GENERAL MODEL FOR MULTI-ITEM LOT SIZING PROBLEM FOR MULTI-SUPPLIERS WITH QUANTITY DISCOUNTS

BY

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This thesis, written by RIO TURNADI under the direction his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN INDUSTRIAL AND SYSTEMS ENGINEERING.**

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Dedicated to my

Dear parents, brother and sisters

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All praise is due to Allah ‘azza wa jalla, We praise Allah ta’ala abundantly, alone with no partners. I bare witness that there is no God except Allah and I bare witness that Muhammad is the slave and the prophet of Allah. May Allah jalla jalaluh exalt his mention, grant peace and send his blessing to the prophet Muhammad Salallahu ‘alaihi wasallam.

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LIST OF ABBREVIATIONS

GA	:	Genetic Algorithm
MSM	:	Modified Silver-Meal
MIP	:	Mixed Integer Programming

ABSTRACT

Full Name : Rio Turnadi

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Lot-sizing is an essential decision in inventory management, in which the optimal product quantity is determined to minimize the total cost of manufacturing. Many models have been developed to formulate and solve this problem. Each model is based on different methods and assumptions. In this work, a general model is proposed for multi-items lot sizing problem with multiple suppliers, quantity discounts, and backordering of shortages. Mixed integer programming (MIP) is used to formulate the problem and obtain the optimum solution for small problems. Due to the large number of variables and constraints in practical problems, the model is too hard for optimal solution. In order to tackle the NP-Hard problem of the model, we proposed two heuristic models. The First one by modifying the Silver-Meal heuristic and the second one by modeling the problem into Genetic Algorithm. Both heuristic methods are shown to be effective and efficient in solving the multi-item lot sizing problem with multi-suppliers and quantity discounts.

ملخص الرسالة

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حجم الصفقة (حجم الدفعة) هو أمر أساسي في إدارة المخازن, حيث يتم به تحديد الإنتاج الأمثل لتقليل تكلفة التصنيع. نماذج عدة تم تطويرها لصياغة و حل مشكلة تحديد عدد الدفعات. كل نموذج منها مبني على طرق و افتراضات مختلفة. في هذا العمل, تم اقتراح نموذج عام لمشكلة حجم الصفقة المتعددة العناصر بها موردين عدة, حسومات (خصومات) في العدد, و بيع مؤجَّل (وعد بتوفير البضاعة المطلوبة) في حالة نفاد البضاعة. تم استخدام برمجة الأعداد المدمجة لصياغة المشكلة و الحصول على الحل المثالي للمشاكل الصغيرة فقط. بسبب المتغيرات و القيود الكثيرة في المشاكل التطبيقية, فإن النموذج يجد صعوبة كبيرة في الحصول على الحل المثالي. للتعامل مع مشكلات "NP-Hard" في النموذج, أقترحنا نموذجين تجريبيين. النموذج التجريبي الأول هو تعديل في نموذج سلفَر-مييل "Silver-Meal" التجريبي, و الثاني هو بنمذجة المشكلة في خوارزمية جينية. كلا النموذجين أظهرا فعالية و كفائية في حل مشكلة حجم الدفعة بموردين عدة و خصميات في الكميات.

CHAPTER 1

INTRODUCTION

Lot sizing is the most important part in inventory management. It has been a becoming area of active research since the first introduction of this problem in 1958. Lot sizing become an interesting topic not only in practical realm but also in academic field. Many mathematical models have been introduced to solve this problem. Because of the complexity of this problem, many issues must be considered in building the model. Some models used multi-objective and other models used single objective optimization, also some models used multi-item while some others used single item lot sizing. Another consideration are related with the wide area of the model, such us considering quantity discount, considering equal or unequal batch in shipment, variability of lead time, and so on.

One of the interesting parts in lot sizing problem is the multiple supplier and each supplier provide its quantity discount. They provide quantity discount in order to attract the buyer for buying the item in large quantity. In this case, buyer will try to consider whether they will order in large quantity in order to get a lower unit-price or only in small amount because of the higher inventory holding cost. There are two types of quantity discount; all-unit quantity discount and incremental quantity discount. Both discounts give different unit cost. Commonly, the buyers have a right to choose from which supplier they will buy the item. Sometimes the buyers buy more than one type of item from one supplier.

Every company tried to satisfy their costumer by providing a reliable service. They tried to make sure that they always have the item in their hand, so that whenever the customers need the item, they can provide it immediately. However, it will make the company should keep the inventory in higher certain level. Keeping more inventories needs more cost, also there will be the opportunity cost. Therefore some companies tried to deal with the customer for backordering the unmet demand. There are several type of backordering condition; full backordering, fixed partial backordering, and full lost sales. Another consideration is related with the limitation of the budget. In most cases, company has a limit in budget they can spend per period for purchasing the items, therefore they cannot purchase all items as much as they want/need. They should consider which item will give more profit, thus will be prioritized. Similarly, the company also has a limitation in storage spaces, therefore they cannot purchase items more than the storage limit.

In the proposed work, we want to develop a model for multi-item lot sizing problem with budget and space constraints under different backordering condition for multi-supplier considering both all-unit quantity discount and incremental quantity discount. Mixed Integer Programming will be used to formulate the model. Since the model become NP-hard, it will take long time to solve it using analytical method, therefore in the proposed work we will also apply heuristic method for solving the problem, which is Genetic Algorithm.

CHAPTER 2

LITERATURE REVIEW

2.1 Lot sizing

In the first introduction of lot sizing, the classical problem was modeled using single item. Okhrin and Richter [1] developed lot sizing model for single item by considering restriction in minimum order quantity. Wagner [2] developed lot sizing model with considering two types of product (perishable product and durable product). The first type of product is strictly to satisfy, no shortage is allowed, in case of unmet demand, we will lose the sale. The second one is more durable, for unmet demand, we can make a shortage and fill it in the next period. An online dynamic model has been obtained for solving the lot sizing problem.

Instead of using either single-item or multi-item in modeling lot sizing problem, either single-level lot sizing problem or multi-level lot sizing problem should be considered. Other considerations that should be included in the model are lead time, transportation, batch, quantity discount and so on. We can also make it as an assumption in order to make the model satisfy the real condition.

Based on the constraint of capacity, there are two types of lot sizing model. The first one is when the capacity constraint including in the lot sizing model, it is usually called by capacitated lot sizing problem. The second one is incapacitated/uncapacitated lot sizing problem which means the model does not consider the capacity constraint, thus capacity of production assumed to be unlimited.

Chu et al. [3] proposed capacitated single item lot sizing model with considering holding, backlogging and outsourcing in cost function. In this model they assumed that holding, backlogging and outsourcing cost are linear.

Since the market become more competitive, some companies try to attract their buyer to buy in large quantity, in order to increase their sales. They introduced a quantity discount per replenishment order. Cha and Moon [4] and Lee et al. [5] developed a model for joint replenishment problem with quantity discounts. Both incremental quantity discount and all-units quantity discount were implemented in the models.

Zhang et al. [6] constructed capacitated lot sizing problem by including transportation cost based on number of containers used per replenishment. Gutierres et al. [7] proposed multi-item lot sizing problem by considering storages capacities. Sadjadi et al. [8] proposed joint lot sizing problem by considering budget constraint.

2.2 Quantity Discount

Quantity discount is one of the interesting parts in economic order quantity (EOQ). It will be provided according to purchasing quantity by the customer, more quantity purchased will result in lower unit price.

Quantity discount will result in saving the cost, however it will affect the holding cost because of purchasing larger quantity. Cha and Moon [4] considered quantity discount for joint replenishment problem under constant demand. They built the model for multi item lot sizing problem and used only all-units quantity discount.

Jung and Klein [9] included quantity discount in cost function for economic order quantity (EOQ) model. The model has been developed for profit maximizing problem and price-depended demand.

Ertogral et al. [10] developed a model for joint vendor-buyer supply chain problem considering transportation cost. The model considered all-unit discount in transportation cost. Ramasesh [11] used price discount as a type of incentive in lot sizing problem. He modeled the problem for a limited-time price reduction for the purchasing cost.

Maiti et al. [12] introduced multi-item lot sizing problem with two-storage (owned storage and rented storage) and considering all-unit discount policy. The model has been built for maximizing the profit function. Shortages are not allowed in the model. Because of the complexity of the model, Genetic Algorithm (GA) has been obtained for solving the model.

Lee et al. [5] obtained both all-unit quantity discount and incremental quantity discount in their lot sizing model. The model consisted of four cost-components; ordering cost, purchasing cost, transportation cost, and holding cost. Drake and Pentico [13] and San-Jose and Garcia-Laguna [14] considered backordering in lot sizing problem using quantity discount.

2.3 Shortages and Backordering

The classical lot sizing problem assumed that all demand should be satisfied. However in fact, sometimes we could not satisfied all demand, and it becomes a shortage. In this case, there are several condition of shortages; full backordering, partial backordering, and full lost sales.

Full backordering means that we can satisfy all un-meet demand at the end of the period/horizon. There are two conditions, either there is a penalty for backordering the demand or there is no penalty. In vice versa, full lost sales means that all un-meet demand cannot be satisfied and as a result, we lost a chance for getting a profit from selling the item. In some researches, they also included other cost for the un-meet demand, for example; lost of goodwill, service level, etc. In partial backordering, we can satisfy some of un-meet demand, while some others will be calculated as lost sales. Many researches have been conducted in partial backordering.

San-Jose et al. [15] developed a model for lot sizing problem using partial backlogging characterized as a behavior of the customer hinges on the waiting time and on the shortage period, which is the longer the waiting time will result more lost sales. In their other models [16] they used exponential partial backordering. Toews et al. [17] obtained backordering which is linearly dependent with delivery time of the item in both EOQ and EPQ models. Aksen [18] proposed uncapacitated lot sizing model by considering loss of customer goodwill. In this model he assumed that unsatisfied demands cannot be backordered and for those unsatisfied demands will lead to loss of customer goodwill. The loss of customer goodwill will be converted in to loss of demand for the next period.

Absi and Kedad-Sidhoum [19] proposed multi-item lot sizing problem considering setup times. In their model, all shortages cannot be backordered, thus will become full lost sales. Yang et al. [20] obtained a research in inventory model with considering both partial backordering and lost sales. They assumed that there is a fraction of backordering demand which is between zero and one ($0 \leq B \leq 1$). The special case, if the fraction is zero, there will be full lost sales, and if the fraction is one, there will be full backordering.

Drake and Pentico [13] developed lot sizing model with partial backordering by including price discount in the model. There is a correlation between the price discount and the backordering percentage. The correlation become a function and included in the objective function of the model.

2.4 Mixed integer programming

Integer programming is used in the model when the variables of the model should be an integer number. In lot sizing problem, most of the models used mixed integer programming model because some of the variables in lot sizing model are integer number (i.e. demand, item quantity, job schedule, etc.) and some of the other variables are not integer number (i.e. cost, time, etc.). Sometimes the mixed integer programming model is combined with the other method, especially when some functions are not linear and become NP-hard.

Tarim and Kingsman [21] combined the mixed integer linear programming with chance-constrained stochastic model to handle both deterministic and the stochastic nature of lot sizing problem. This model considered service level as constraint in the formulation.

Eksioglu [22] developed a model for multi-mode lot sizing problem using multi suppliers, different transportation scheme, and considering different setup cost function. He formulated the model as a network problem and solved it using MIP. Haugland et al. [23] developed langrangian relaxation techniques based on a straightforward mixed integer programming formulation and demonstrated how this can provide strong bounds on the maximum flow with minimum lot size.

Sung and Maravelias [24] proposed multi item lot sizing problem on EPQ using single stage process for a longer setup times. They obtained MIP for solving the model. Clark and Clark [25] developed lot sizing model on EPQ using parallel machine. The model considered sequence dependent set up times. They obtained MIP to formulate the model and solved it by Fixed and Relax approach.

Absi and Kedad-Sidhoum [19] obtained MIP for multi item lot sizing problem. The model considered setup time under the assumption that unmet demand cannot be backordered, thus became a shortage cost.

Gao et al. [26] studied lot sizing problem under the condition where there are two types of setup cost; major setup cost for a batch production, and minor setup cost for producing each component. They formulated the model using MIP formulation and solved the model by relaxing it using Linear Programming (LP) relaxation.

Rezaei and Davoodi [27] used multi-objective mixed integer programming in lot sizing problem considering supplier selection. Three objectives of the model are total cost, quality level, and service level, thus the model became non-linear problem. Since the model is NP-hard problem, they used MIP just for formulating the model, then for solving the model they obtained heuristic method.

Woarawichai et al. [28] developed lot sizing problem with supplier selection by considering storage space and budget constraints in the model. MIP has been obtained for formulating and solving the model by using LINGO as optimization software.

2.5 Genetic Algorithm

Most of the lot sizing models are NP-hard problem which are very complicated and difficult to solve using analytical methods, it will take long calculation time, therefore in most cases researchers combined analytical method with heuristic method for solving the model. The most commonly used of the heuristic method in lot sizing problem is Genetic Algorithm (GA), because of several reasons:

- Can be used for any objective function
- Useful for not only single objective but also for multi objective optimization
- Fast calculation time and nearest to the optimal solution
- Easy to implement and to program in any programming language

Gaafar [29] obtained genetic algorithm in dynamic lot sizing problem using batch ordering. He also compared the performance of GA with the modified silver-meal (MSM) heuristic. The result showed that the performance of GA is better than the performance of MSM.

Rezaei and Davoodi [27] constructed multi-objective lot sizing problem considering supplier selection. The objectives of the model are maximizing quality and service level and minimizing the cost. They obtained GA for solving the model. Because of the multi-objective in the model, they used the modified GA which is NSGA II (Non-dominated Sorting Genetic Algorithm-II) as an alternative for pareto optimal solutions.

Pasandideh et al. [30] developed a model in inventory problem for multi-item by considering multi-constraints. The model considered backordering for shortages. GA has been used for finding the optimal order quantity and backorder level in order to minimize the total cost of inventory, since the model built for vendor managed inventory problem.

Lee et al. [5] used GA in solving an integrated lot sizing problem. The model worked for single item lot sizing problem considered supplier selection while each supplier provided their own discount scheme. The objective of the model is to minimize the total cost which is summation of; ordering cost, unit cost, transportation cost, and holding cost. The result showed that GA more effective than MIP in case of calculation time.

CHAPTER 3

MODEL FORMULATION

3.1 General Model Formulations

We construct the model with the following assumption:

- Demands are known for each item and each period.
- Each item can be ordered at most once per period.
- Each unit has quantity discount, depends on the supplier's quantity policy, using both all-unit discount and incremental discount.
- Lead time is known and constant, the inventory will be received at the beginning of each period.
- Both inventory and shortage are allowed in any period.
- Cost of inventory during period is known and constant (not depended on unit price)
- The shortage cost during a period is known and constant (not depended on unit price).
- The transportation cost will be calculated per vehicle. a vehicle can be used for more than one type of item, it depends on the capacity of the vehicle, in this model capacity of the vehicle based on the volume of the container (for instance, item A, B, and C can be carried out by 1 vehicle, however the total volume of all those items should not exceed the capacity (container's volume of the vehicle)).

- For the backordered items, the items are forwarded to the customer once they are received
- Finite and know planning horizon (T).
- Initial values for both inventory and shortage are zeros.
- All shortages should be fulfilled at the end of horizon.

Generally, the initial model consists of an objective function which is minimizing the total cost subject to some constraints.

Objective Function:

Minimize → Total Cost =

Ordering Cost + Purchasing Cost + Transportation Cost + Holding Cost + Shortage Cost

Subject to, Constraints:

- Demand per period
- Budget Limit
- Storage Limit
- Constraints related to MIP condition

Here are all notations for the model.

Notations

Indices:

t Planning period (t = 1, 2,…, T).

i Supplier (i = 1, 2,…, I). 1 to i' for all-units discounts, i' + 1 to I for incremental discounts.

j Item (j= 1, 2,…, J). 1 to j' for all-units discounts, j' + 1 to J for incremental discounts.

k Price break of discount policy (k = 1, 2,…, K).

Parameters:

d_{jt} Demand in period t for item j.

h_j Holding cost for item j (the same holding cost for each period).

l_j Shortage cost for item j (the same shortage cost for each period).

o_{ij} Ordering cost for item j from supplier i.

c_i Container's volume of the vehicle from supplier i.

v_j Volume unit of item j.

s_i Transportation cost per vehicle from supplier i.

w_t Budget for each replenishment in each period t.

y Storage capacity.

p_{ijk} Unit purchase cost for item j from supplier i with price break k under all-units quantity discounts.

ps_{ijk} Unit purchase cost for item j from supplier i with price break k under all-units quantity discounts.

$\hat{p}_{ijkt}$ Average unit purchase cost for item j from supplier i in period t with purchase quantity under price break k under incremental quantity discounts.

q_{ijk} The upper bound quantity for item j of supplier i with price break k.

Variables:

TC Total cost for all planning period.

QP_{ijt} Purchase quantity for item j from supplier i in period t.

CP_{ijt} Purchase cost for one unit item j based on the discount schedule of supplier i with order quantity QP_{ijt} in period t.

CI_{jt} Inventory cost for item j in period t.

CS_{jt} Shortage cost for item j in period t.

N_{it} Number of transportations (vehicles) from supplier i in period t.

IB_{jt} Beginning inventory level for item j in period t.

IE_{jt} Ending inventory level for item j in period t.

SB_{jt} Beginning shortages level for item j in period t.

SE_{jt} Ending shortages level for item j in period t.

AB_{jt} A binary variable for beginning inventory, set equal to 1 if there is inventory, and 0 if there is no inventory for item j, at the beginning of period t.

AE_{jt} A binary variable ending inventory, set equal to 1 if there is inventory, and 0 if there is no inventory for item j, at the end of period t.

BB_{jt} A binary variable beginning shortages, set equal to 1 if there is shortage, and 0 if there is no shortage for item j, at the beginning of period t.

BE_{jt} A binary variable ending shortages, set equal to 1 if there is shortage, and 0 if there is no shortage for item j, at the end of period t.

F_{ijt} A binary variable, set equal to 1 if a purchase for item j is made, and 0 if no purchase for item j is made, from supplier i in period t.

U_{ijtk} A binary variable, set equal to 1 if a certain quantity for item j is purchased, and 0 if no purchase for item j is made, from supplier i with price break k in period t.

M Sufficiently large numbers.

Fig. 1 represent the basic idea of the model, assumed that this for item j (since the model for multi-item). It gives the representation of how the model works. At the beginning of period (period 1), there is no either inventory or shortage (I_{j0} = 0 and S_{j0} = 0), so for the period 1, the beginning inventory is equal to the total purchase quantity from all suppliers. For the next period, we consider either there is inventory or shortage from previous period, or both level are zero. For example, in the picture, for period 2, there is shortage from period 1 and at the beginning of period 2 we order some amount of item, thus the total inventory level at the beginning of period 2 is equal to total quantity purchased minus shortage from period 1. In general, the beginning inventory is given by:

$$IB_{jt} - SB_{jt} = IE_{j(t-1)} - SE_{j(t-1)} + \sum_{i=1}^{I} QP_{ijt}$$

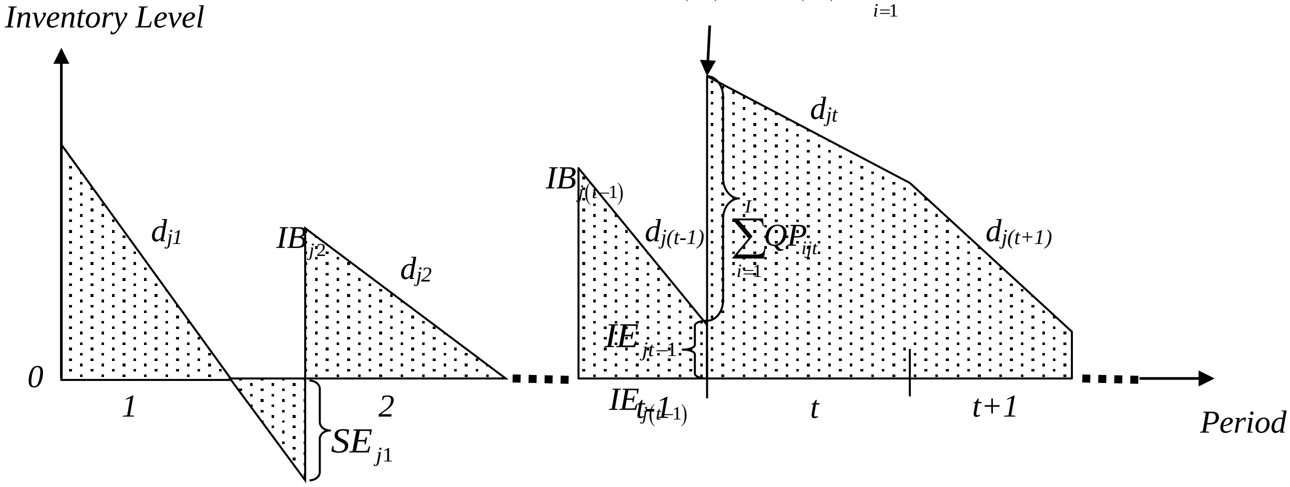

Figure 1 *Graphical representation of system replenishment*

3.2 Model formulation and related cost

The lot sizing problem in this paper will be formulated using Mixed Integer Programming (MIP).

Eq. (1) is used for calculating the cost for each replenishment of item *j* from supplier *i* for period *t*.

$$Ordering\ cost = CO = \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{i=1}^{I} o_{ij} \times F_{ijt} \tag{1}$$

Eq. (2) is used for calculating the unit cost of item j from supplier i for period t follow the discount scheme provided by the supplier for all unit quantity discount.

$$Purchase\,cost = CP_{ijt}$$

$$CP_{ijt} \geq \sum_{k=1}^{K} p_{ijk} \times QP_{ijt} \tag{2}$$

where

$$q_{ij(k-1)} + M \times \left(U_{ijtk} - 1\right) < QP_{ijt} \leq q_{ijk} + M \times \left(1 - U_{ijtk}\right)$$

$$\sum_{k=1}^{K} U_{ijtk} = F_{ijt}$$

Eq. (3) is used for calculating the transportation cost for each replenishment of all items purchased from supplier i for period t.

$$Transportation\,cost = CT = s_i \times N_{it} \tag{3}$$

where

$$\sum_{j=1}^{J} v_j \times QP_{ijt} \leq N_{it} \times c_{it}$$

The average inventory for each item in each period is one-half of the total expected ending inventory from previous period and the beginning inventory for such item in that period.

There are three possible cases (combinations) for beginning and ending inventory levels as shown in Figure 5.

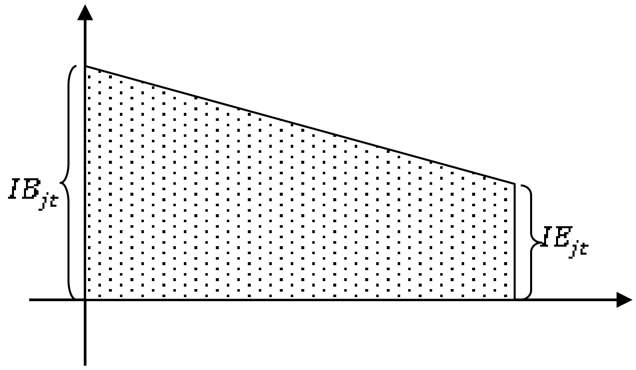

Figure 2 *Case I of Inventory problem: Inventory for both beginning and ending of period*

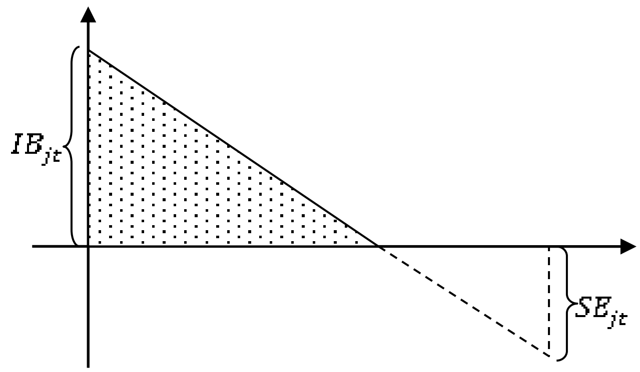

Figure 3 *Case II of Inventory problem: Inventory for beginning and shortage at the end of the period*

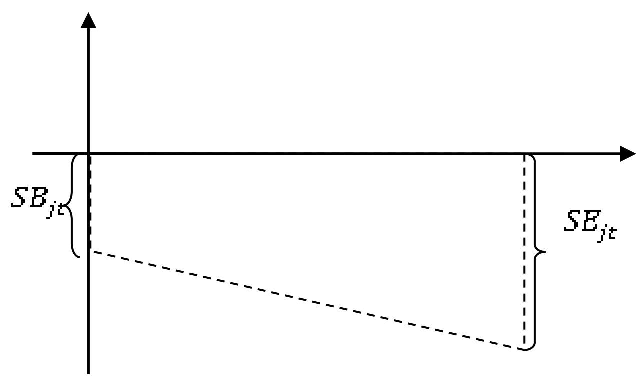

Figure 4 *Case III of Inventory problem: Shortage for both beginning and ending of period (no inventory)*

$$Holding\,cost = CI_{jt}$$

$$CI_{jt} \geq \frac{h_j}{2}\left(IB_{jt} + IE_{jt}\right) - M \times BE_{jt} \tag{4}$$

$$CI_{jt} \geq \frac{h_j\left(IB_{jt}\right)^2}{2\left(d_{jt}\right)} - M \times \left(1 - BE_{jt} + BB_{jt}\right) \tag{5}$$

where

$AB_{jt} + BB_{jt} = 1$

$AE_{jt} + BE_{jt} = 1$

$IB_{jt} \leq M \times AB_{jt}$

$SB_{jt} \leq M \times BB_{jt}$

$IE_{jt} \leq M \times AE_{jt}$

$SE_{jt} \leq M \times BE_{jt}$

The shortages cost is defined as follows. As mentioned before, shortage will happen only if the ending inventory level from the previous period is less than zero, which means we have shortage from the previous period.

There are three possible cases (combinations) for beginning and ending shortage levels as shown in Figure 5, Figure 6 and Figure 7.

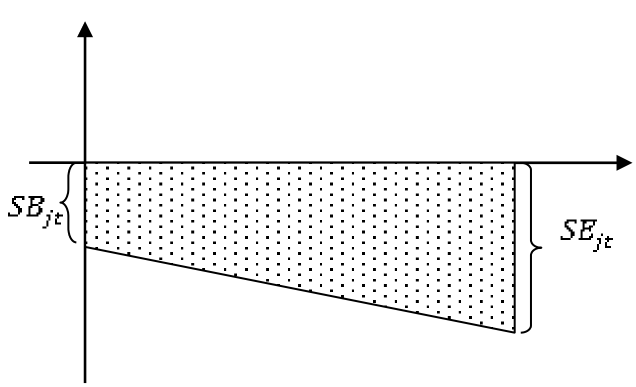

Figure 5 *Case I of Shortage problem: Shortage for both beginning and ending of period*

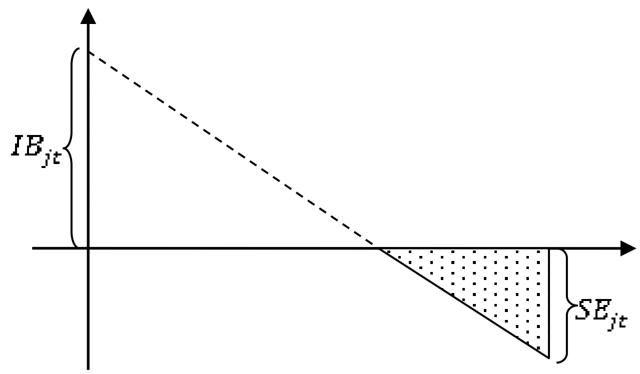

Figure 6 *Case II of Shortage problem: Inventory for beginning and shortage at the end of the period*

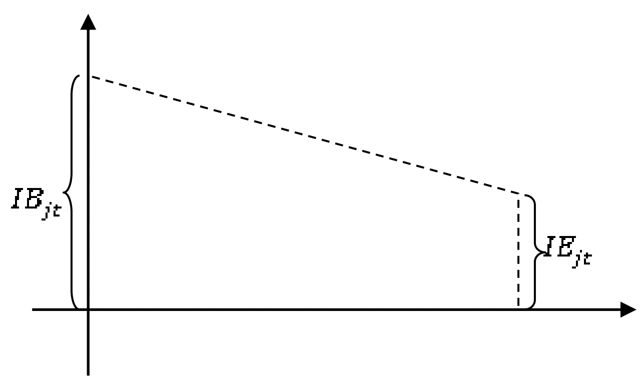

Figure 7 *Case III of Shortage problem: Inventory for both beginning and ending of period (no shortage)*

The average shortages for each item in each period is one-half of the total ending shortage from previous period for such item in that period.

$$Shortages\,cost = CS_{jt}$$

$$CS_{jt} \geq \frac{l_j}{2}\left(SB_{jt} + SE_{jt}\right) - M \times AB_{jt} \tag{6}$$

$$CS_{jt} \geq \frac{l_j \left(SE_{jt}\right)^2}{2\left(d_{jt}\right)} - M \times \left(1 - AB_{jt} + AE_{jt}\right) \tag{7}$$

where

$$AB_{jt} + BB_{jt} = 1$$

$$AE_{jt} + BE_{jt} = 1$$

$$IB_{jt} \leq M \times AB_{jt}$$

$$SB_{jt} \leq M \times BB_{jt}$$

$$IE_{jt} \leq M \times AE_{jt}$$

$$SE_{jt} \leq M \times BE_{jt}$$

3.3 MIP Formulation

Since we have a clear and deterministic objective function along with sets of constraints, and because of some constraints should be integer, some of them should be binary, and others no restriction as long as the value is non-negative, and one of the value should be unrestricted sign (can be any real number, both positive and negative are allowed), therefore MIP is the best choice for solving the model.

Here are the formulations of the lot sizing model:

The objective function, minimizing total cost (cost of ordering the item + cost of purchasing + cost of transportation + cost of shortage + cost of holding inventory)

$$Minimize TC = \sum_{t=1}^{T} \left(\sum_{j=1}^{J} \left(\sum_{i=1}^{I} \left(o_i \times F_{ijt} + CP_{ijt} \right) + CI_{jt} + CS_{jt} \right) + \sum_{i=1}^{I} s_i \times N_{it} \right) \tag{8}$$

Eq. (9), constraint for the ending inventory.

$$IE_{jt} - SE_{jt} = IB_{jt} - SB_{jt} - d_{jt} \tag{9}$$

Eq. (10), constraint for the beginning inventory level.

$$IB_{jt} - SB_{jt} = IE_{j(t-1)} - SE_{j(t-1)} + \sum_{i=1}^{I} QP_{ijt} \tag{10}$$

Eq. (11), constraint related with transportation for all suppliers.

$$\sum_{j=1}^{J} v_j \times QP_{ijt} \leq N_{it} \times c_{it} \tag{11}$$

Eq. (12), constraint for amount of quantity purchased.

$$QP_{ijt} \leq M \times F_{ijt} \tag{12}$$

Eq. (13), constraint related with all unit quantity discount from supplier 1 up to supplier i'. purchase cost per unit item, which is given by:

$$CP_{ijt} \geq \sum_{k=1}^{K} p_{ijk} \times QP_{ijt} \tag{13}$$

Eq. (14), constraint related with all unit quantity discounts from supplier 1 up to supplier i'. Here is the function for lower bound and upper bound of the quantity.

$$q_{ij(k-1)} + M \times \left(U_{ijtk} - 1\right) < QP_{ijt} \leq q_{ijk} + M \times \left(1 - U_{ijtk}\right) \tag{14}$$

Eq. (15), constraint related with all unit quantity discounts from supplier 1 up to supplier i'. this function will make sure only one of the quantity discount for each price break can be purchased.

$$\sum_{k=1}^{K} U_{ijtk} = F_{ijt} \tag{15}$$

Eq. (16), constraint related with incremental quantity discount from supplier $i'+1$ up to supplier I. The following function is the constraint for purchase cost per unit item using incremental discount.

$$CP_{ijt} \geq \sum_{k=1}^{K} \hat{p}_{ijtk} \times QP_{ijt} \tag{16}$$

Eq. (17), constraint related with incremental quantity discount.

$$\hat{p}_{ijtk} = p_{ijk} + \frac{\alpha_{ijk}}{QP_{ijt}},\ QP_{ijt} \neq 0 \tag{17}$$

Eq. (18)

$$\alpha_{ijk} = \sum_{k'=1}^{k} q_{ijk'} \times \left(p_{ijk'-1} - p_{ijk'} \right) \tag{18}$$

$$QP_{ijt} \leq M \times U_{ijtk} \tag{19}$$

Eq. (20), constraint binary variable for purchasing item.

$$F_{ijt} \in \{0,1\} \tag{20}$$

Eq. (21), constraint binary variable for purchasing item in price break k.

$$U_{ijtk} \in \{0,1\} \tag{21}$$

Eq. (22) constraint binary variable for beginning inventory.

$$AB_{jt} \in \{0,1\} \tag{22}$$

Eq. (23) constraint binary variable for beginning shortage.

$$BB_{jt} \in \{0,1\} \tag{23}$$

Eq. (24) constraint binary variable for ending inventory.

$$AE_{jt} \in \{0,1\} \tag{24}$$

Eq. (25) constraint binary variable for beginning shortage.

$$BE_{jt} \in \{0,1\} \tag{25}$$

Eq. (26) constraint related with inventory and shortage at the beginning and ending of period.

$$AB_{jt} + BB_{jt} = 1 \tag{26}$$

$$AE_{jt} + BE_{jt} = 1 \tag{27}$$

$$IB_{jt} \leq M \times AB_{jt} \tag{28}$$

$$SB_{jt} \leq M \times BB_{jt} \tag{29}$$

$$IE_{jt} \leq M \times AE_{jt} \tag{30}$$

$$SE_{jt} \leq M \times BE_{jt} \tag{31}$$

All shortages should be fulfilled at the end of the horizon. Hence the total quantity purchased should be equal to the total demand during the horizon;

$$\sum_{i=1}^{I}\sum_{t=1}^{T} QP_{ijt} = \sum_{t=1}^{T} d_{jt} \tag{32}$$

Eq. (33) constraint related to budget for each replenishment

$$\sum_{i=1}^{I}\sum_{j=1}^{J} CP_{ijt} <= w_t \tag{33}$$

Eq. (33) constraint related to capacity of the storage

$$\sum_{j=1}^{J} v_j \times IB_{jt} <= y \tag{34}$$

All variables are nonnegative.

3.4 Relaxing the Model

The proposed MIP model is hard to solve within reasonable computational time because of some non-linearity on the constraints and due to the large number of variables and constraints.

It is possible to reduce the number of constraints and variables by relaxing the assumption of the model. For the initial MIP model, it is allowed to order from different supplier per order, here we restrict the model by forbidding order from different suppliers per order, only from one supplier is allowed to order, the one which give the least total cost.

We end up with the following MIP formulation:

The objective function, minimizing total cost (cost of ordering the item + cost of purchasing + cost of transportation + cost of shortage + cost of holding inventory)

$$Minimize\,TC = \sum_{t=1}^{T}\left(\sum_{j=1}^{J}\left(\sum_{i=1}^{I}\left(o_i \times F_{it} + CP_{ijt}\right) + CI_{jt} + CS_{jt}\right) + \sum_{i=1}^{I} s_{it} \times N_{it}\right) \tag{35}$$

Eq. (36), constraint for the ending inventory, remains the same with the original formulation.

$$IE_{jt} - SE_{jt} = IB_{jt} - SB_{jt} - d_{jt} \tag{36}$$

Eq. (37), constraint for the beginning inventory level, remains the same with the original formulation.

$$IB_{jt} - SB_{jt} = IE_{j(t-1)} - SE_{j(t-1)} + \sum_{i=1}^{I} QP_{ijt} \tag{37}$$

Eq. (38), constraint related with transportation for all suppliers, remains the same with the original formulation.

$$\sum_{j=1}^{J} v_j \times QP_{ijt} \le N_{it} \times c_{it} \tag{38}$$

Eq. (39), constraint for amount of quantity purchased.

$$QP_{ijt} \le M \times F_{it} \tag{39}$$

Eq. (40), constraint related with all unit quantity discount from supplier 1 up to supplier i'. purchase cost per unit item. It remains the same with the original formulation.

$$CP_{ijt} \ge \sum_{k=1}^{K} p_{ijk} \times QP_{ijt} \tag{40}$$

Eq. (41), constraint related with all unit quantity discounts from supplier 1 up to supplier i'. Here is the function for lower bound and upper bound of the quantity.

$$q_{ij(k-1)} + M \times \left(U_{itk} - 1\right) < QP_{ijt} \le q_{ijk} + M \times \left(1 - U_{itk}\right) \tag{41}$$

Eq. (42), constraint related with all unit quantity discounts from supplier 1 up to supplier i'. this function will make sure only one of the quantity discount for each price break can be purchased.

$$\sum_{k=1}^{K} U_{itk} = F_{it} \tag{42}$$

Eq. (43), constraint related with incremental quantity discount from supplier *i′+1* up to supplier *I*. The following function is the constraint for purchase cost per unit item using incremental discount.

$$CP_{ijt} \geq X + p_{ijk} \times QP_{ijt} - M \times (1 - U_{itk}) \tag{43}$$

$$\begin{aligned} X = {} & p_{ij1}q_{ij1} + p_{ij2}q_{ij2} + \ldots + p_{ijk-1}q_{ijk-1} \\ & - p_{ijk}q_{ijk-1} - \ldots - p_{ij2}q_{ij1} \end{aligned} \tag{44}$$

$$QP_{ijt} \leq M \times U_{itk} \tag{45}$$

Eq. (46), constraint binary variable for purchasing item.

$$F_{it} \in \{0,1\} \tag{46}$$

Eq. (47), constraint binary variable for purchasing item in price break *k*.

$$U_{itk} \in \{0,1\} \tag{47}$$

Eq. (48) constraint binary variable for beginning inventory.

$$AB_t \in \{0,1\} \tag{48}$$

Eq. (49) constraint binary variable for beginning shortage.

$$BB_t \in \{0,1\} \tag{49}$$

Eq. (50) constraint binary variable for ending inventory.

$$AE_t \in \{0,1\} \tag{50}$$

Eq. (51) constraint binary variable for beginning shortage.

$$BE_t \in \{0,1\} \tag{51}$$

Eq. (52) constraint related with inventory and shortage at the beginning and ending of period.

$$AB_t + BB_t \leq 1 \tag{52}$$

$$AE_t + BE_t \leq 1 \tag{53}$$

$$IB_{jt} \leq M \times AB_t \tag{54}$$

$$SB_{jt} \leq M \times BB_t \tag{55}$$

$$IE_{jt} \leq M \times AE_t \tag{56}$$

$$SE_{jt} \leq M \times BE_t \tag{57}$$

All shortages should be fulfilled at the end of the horizon. Hence the total quantity purchased should be equal to the total demand during the horizon.

$$\sum_{i=1}^{I}\sum_{t=1}^{T} QP_{ijt} = \sum_{t=1}^{T} d_{jt} \tag{58}$$

Eq. (59) constraint related to budget for each replenishment, remains the same with the original formulation.

$$\sum_{i=1}^{I}\sum_{j=1}^{J} CP_{ijt} <= w_t \tag{59}$$

Eq. (60) constraint related to capacity of the storage, remains the same with the original formulation.

$$\sum_{j=1}^{J} v_j \times IB_{it} <= y \tag{60}$$

All variables are nonnegative.

3.5 Modified Silver-Meal Heuristic

The complexity of the above MIP model is due to the large number of variables and constraints as well, which are multiplied by the number of periods, suppliers and quantity discount schemes. Due to this large number of variables, many of which are integer, the model is too difficult to solve for realistic-sized lot-sizing problems. Therefore, an efficient heuristic method was developed for solving the problem by modifying the well-known Silver-Meal heuristic. This heuristic is capable of producing near-optimal solutions in reasonable computational times. Here are the steps of the modified Silver-Meal (MSM) heuristic algorithm:

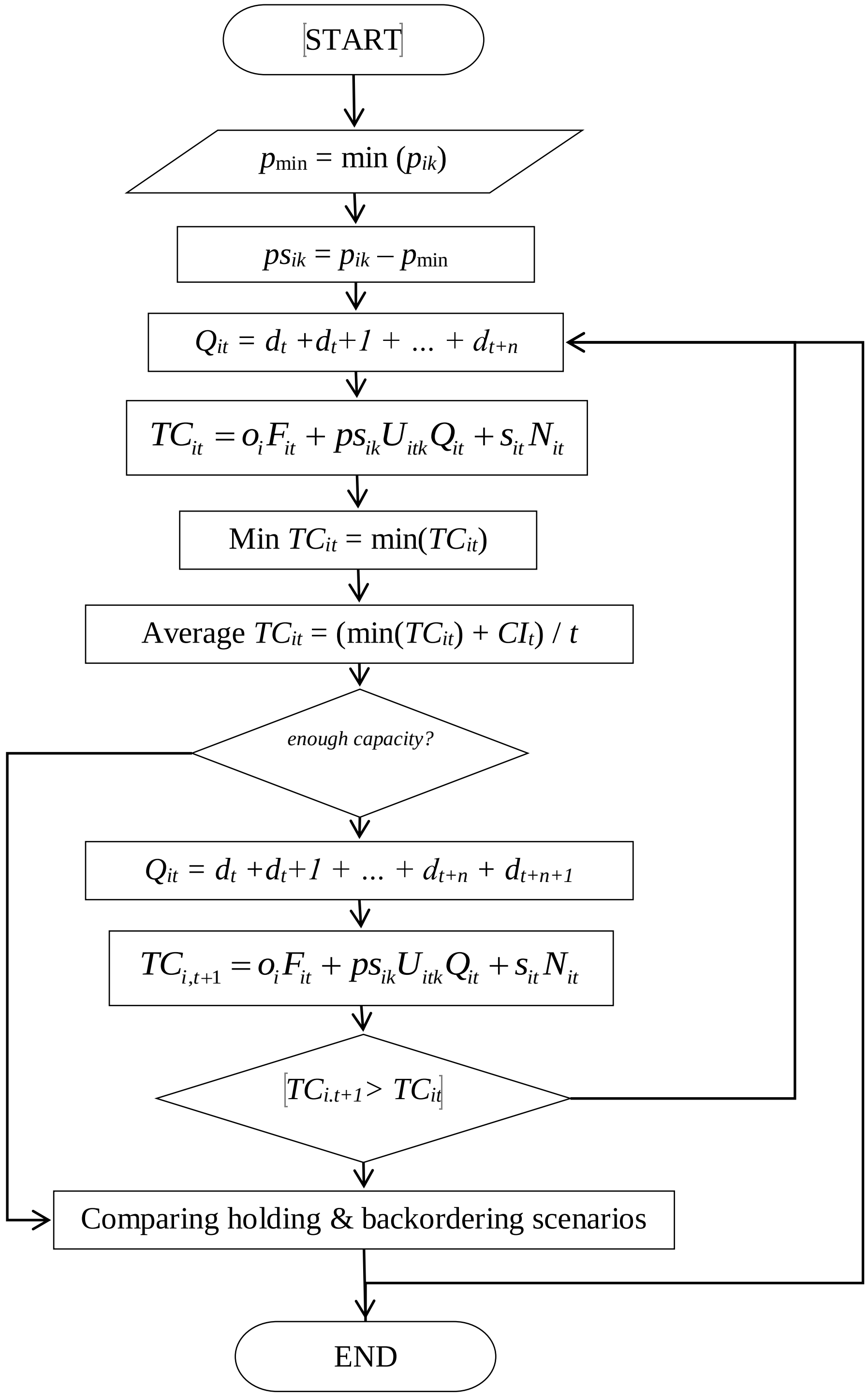

Figure 8 *MSM Steps for Multi-Item lot sizing problem with quantity discount*

Step 1. Choose the minimum cost among all the unit costs from all suppliers and all price breaks. Highest cost = $p_{\min}$ = min(p_{ik}).

Step 2. Since the unit cost depends on number of unit purchased (purchased quantity), the purchased cost will also be considered in our modified silver-meal heuristic. The total purchasing cost tends to be very large compared to other costs. Therefore, only purchase cost differences are included to prevent the total purchasing cost from becoming the overwhelming factor in lot-sizing decisions. From all unit costs provided by all suppliers in all price breaks, subtract the highest unit cost found in step 1. Subtracted cost = ps_{ik} = $p_{ik} - p_{\min}$. This subtracted cost will be either zero or a negative value, indicating the savings in unit purchase cost if we order from a given supplier.

Step 3. Set $n = 0$. Assume the current time period is t, and the order Qit covers periods t, …, $t + n$. Total cost for each supplier is the sum of subtracted purchasing cost, ordering cost, and transportation cost, if a purchase is made from the given supplier. Total cost =

$$TC_{it} = o_i F_{it} + ps_{ik} U_{itk} Q_{it} + s_{it} N_{it} \tag{35}$$

Step 4. Based on the results in step 3, choose the supplier with the least cost (TC_{it}).

Step 5. Add holding cost for periods t, …, $t + n$ to the minimum (TC_{it}) of step 4 and divide it by the number of periods (n + 1) to determine the average cost per period.

Step 6. Repeat steps 3 to 5 after including the demand for the next period (i.e. let $n = n + 1$) in the current order. Keep adding the next periods, one by one, as long as the minimum average cost (of step 5) is decreasing. Stop as soon as the average cost per period begins to increase.

Step 7. As the minimum average cost is increasing we will stop the current iteration and we will move to the next iteration. However, before moving to the next iteration (order), confirm the stopping decision by comparing two options:

- Stopping immediately and proceeding to the next order.
- Adding one more period to cover in the current order.

Adding one more period, compare the additional holding cost with the savings resulting from buying larger quantities. Add this period if the holding cost increase is less than the savings, otherwise, stop the iteration.

Step 8. The algorithm stops the current iteration (order) according to two stopping criteria, whichever comes first: (i) the average cost per period is increasing, or (ii) capacity constraints (11) are exceeded.

Step 9. After covering all demands up to and including current period n, we check all the possibilities for holding and backordering condition. Supposed that in the first iteration, it will cover demands for period 1 up to 3 (we stop iteration at period 4 since the minimum average cost is increasing). In this case there are 3 possibilities;

- Order in period 1, keep the inventories for period 2 and period 3
- Order in period 2, backordering for period 1 and inventory for period 3
- Order in period 3, backordering for period 1 and period 2

For those 3 possibilities, we calculate the average cost, and chose the scenario that gives minimum average cost.

Step 10. Restart the algorithm from time period (i.e. let $t = t + n + 1$), going again through steps 3 to 9.

3.6 Genetic Algorithm (GA) model

Since the analytical model (MIP using LINGO) is not effective to solve the problem, because of long calculation time, even for a simple case, also by considering the lack of Silver-Meal algorithm, therefore another heuristic method will be used for solving the problem. In this work we used Genetic Algorithm (GA) using MATLAB. Here is the procedure of our GA:

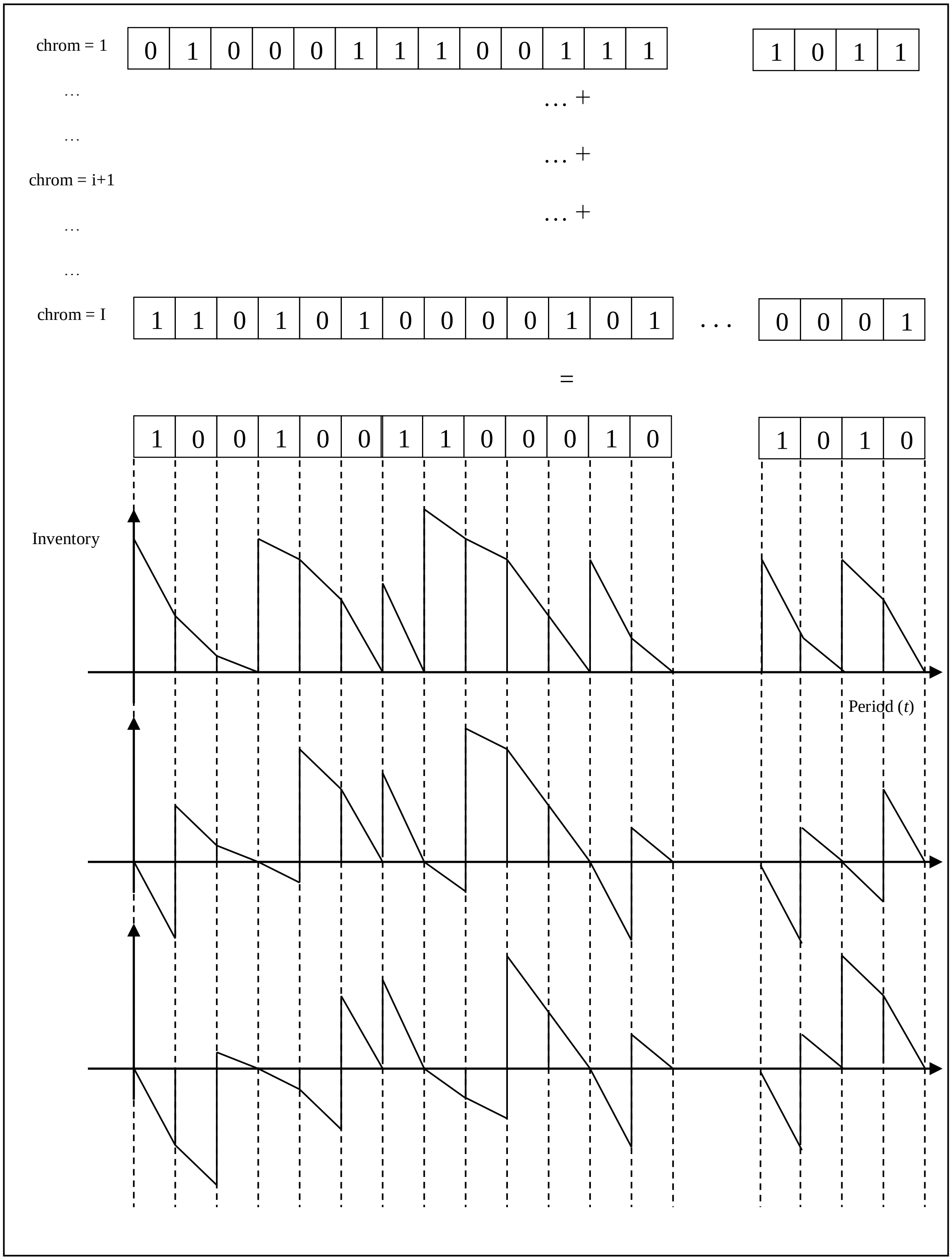

Figure 9 *GA Procedures for Multi-Item lot sizing problem with quantity discount*

Here are the steps for our GA procedure:

Step 1. Coding scheme.

Coding scheme is the way to generate a new population. The strategy is that maximum only 1 order is allowed in each period and it is always at the beginning of the period. The replenishment decision will follow the random integer number (0 or 1). For the given item *j* and period *t*, the random number will code as F_t which means if F_t is equal to 1 we order at period *t*. Note that in our GA model, if the order is made on the specific period, only one supplier is chosen which is the minimum cost for overall items.

The total quantity of each item for the current period is given by:

$$QP_{jt} = F_t d_{jt} + F_t \begin{bmatrix} (1-F_{t+1})d_{jt+1} + (1-F_{t+1})(1-F_{t+2})d_{jt+2} \\ +...+(1-F_{t+1})(1-F_{t+2})\ldots(1-F_T)d_{JT} \end{bmatrix} \tag{61}$$

and the quantity related to the supplier is given by:

$$QP_{jt} = \sum_{i=1}^{I} QP_{ijt} \times F_{it} \tag{62}$$

$$F_t = \sum_{i=1}^{I} F_{it} \tag{63}$$

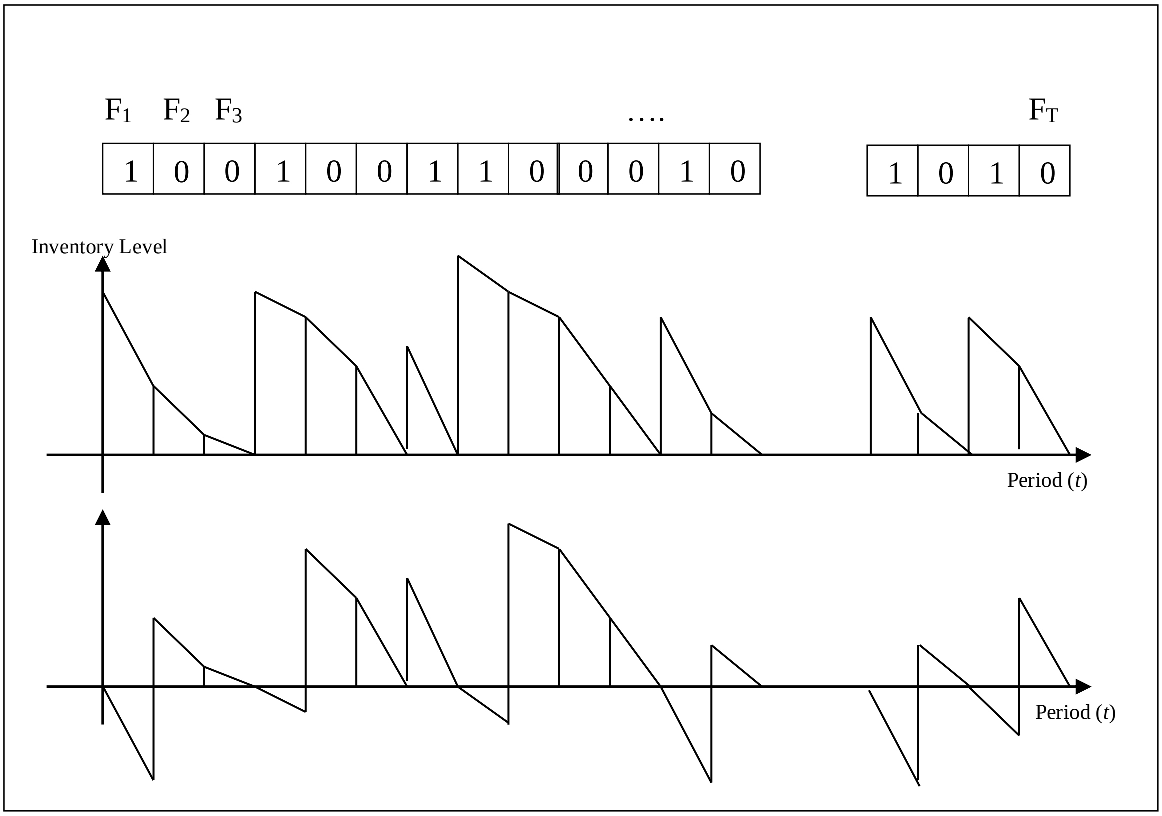

Figure 10 *Coding scheme and replenishment strategy*

Step 2. Initial population of chromosomes.

There is 1 population for each generation. The population contains several numbers of chromosomes. The chromosome will represent the replenishment strategy. Each chromosome contains several numbers of genes. The gene represents the number of period, for example; if the horizon is for 10 periods, the chromosome will contain 10 genes. All genes are based on random binary variable.

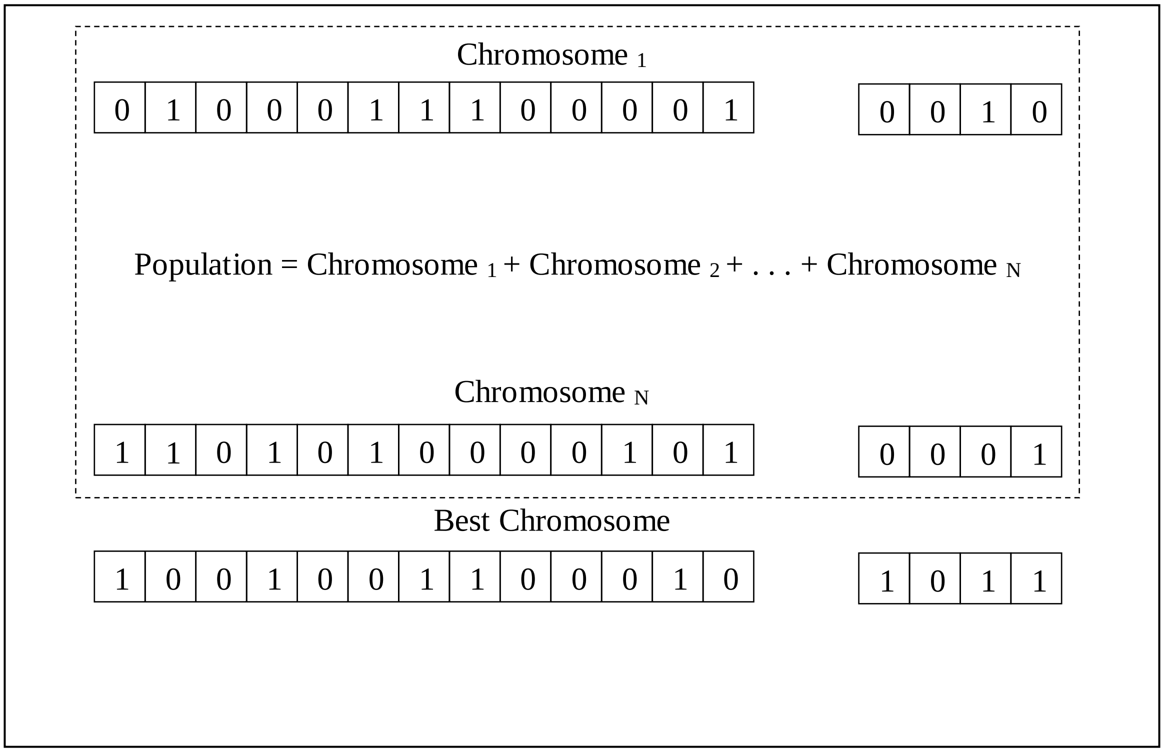

Figure 11 *Population and chromosomes*

Step 3. Fitness function.

The fitness function is the minimum total cost for each chromosome. The formula of the minimum total cost is as given in the MIP model. The total cost is including purchasing cost, ordering cost, transportation cost, and holding cost. For the current chromosome we consider shortage by calculating each possibility of making shortage. Supposed that the current chromosome is 1 0 0 1 0 0 0 1 0 0, the first order will cover demands for period 1 up to 3. In this case there are 3 possibilities;

- Order in period 1, keep the inventories for period 2 and period 3
- Order in period 2, backordering for period 1 and inventory for period 3
- Order in period 3, backordering for period 1 and period 2

For those 3 possibilities, we calculate the average cost, and chose the scenario that gives minimum average cost.

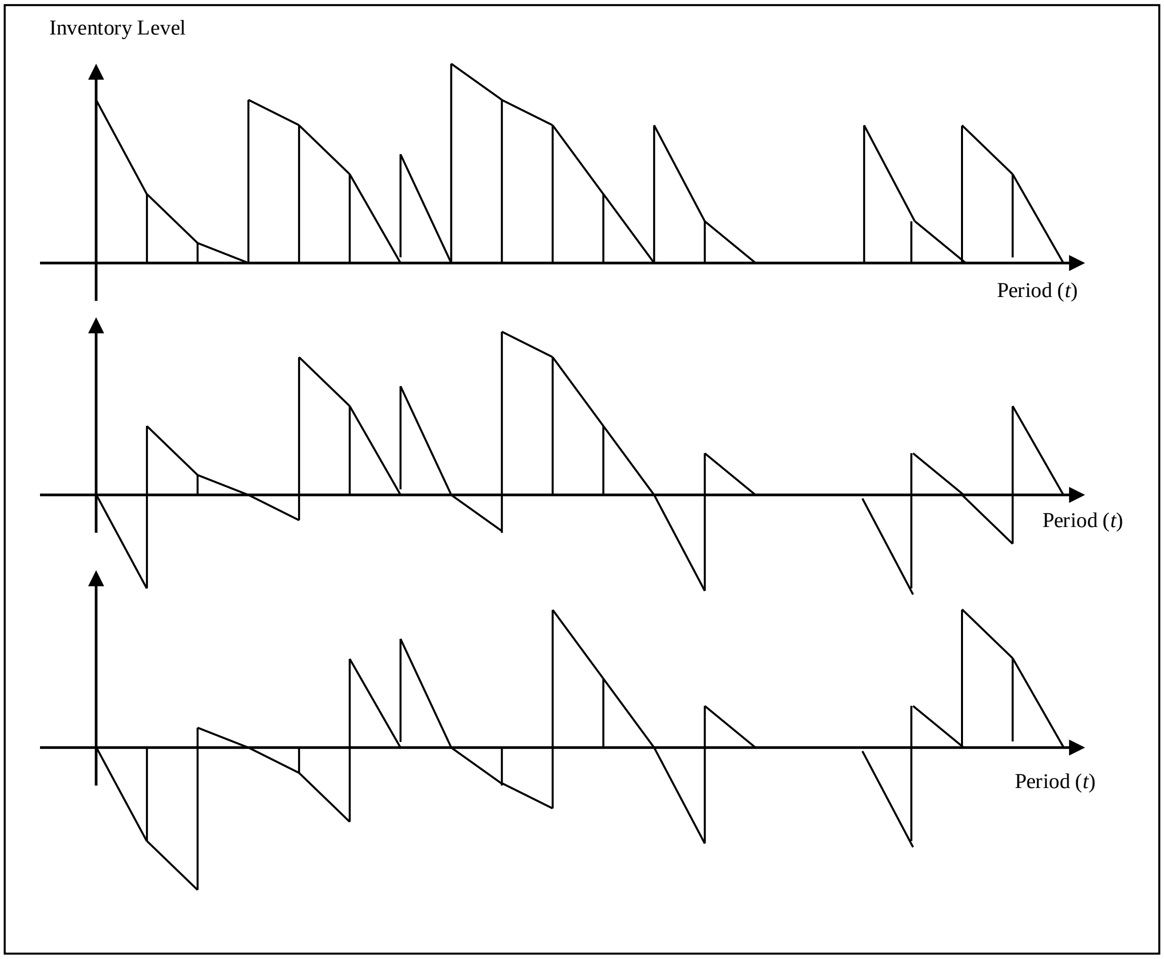

Figure 12 *Incorporating shortage in fitness function*

Step 4. Crossover operation.

We do simple crossover operation which is two-cut crossover. We choose two chromosomes randomly and each chromosome is divided into two parts. The first part of chromosome 1 combine with the second part of chromosome 2, it becomes the first new chromosome. The second part of chromosome 1 combine with the first part of chromosome 2, it becomes the second new chromosome.

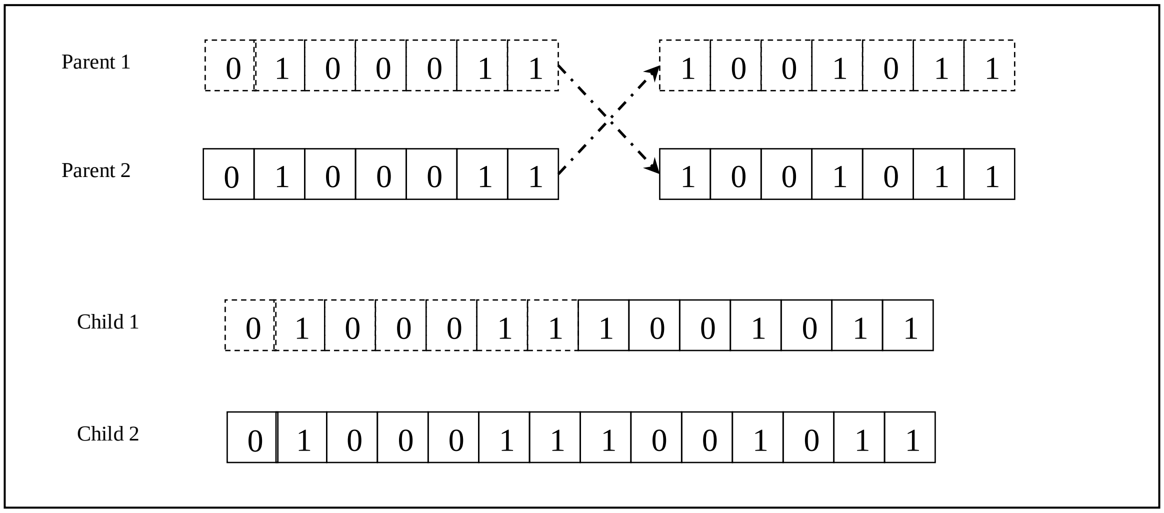

Figure 13 *Crossover operation*

Step 5. Mutation operator.

We do mutation by changing the randomly selected gene from 1 to 0 or from 0 to 1. This mutation process is to maintain diversity of the chromosomes.

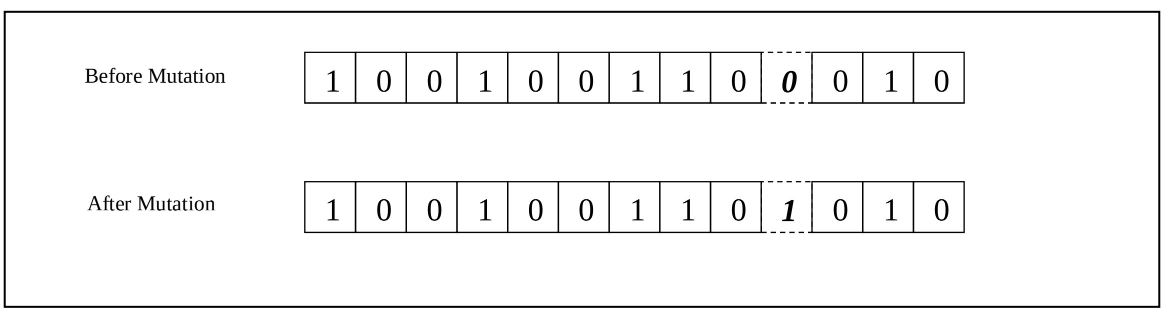

Figure 14 *Mutation operator*

Step 6. Selection of subsequent population.

New population is the sets of chromosomes after crossover and mutation. We generate a new population for every generation. Every chromosome in a new population will be rank based on their fitness value from fitness function.

Step 7. Termination.

The processes of crossover, selection and replacement are repeated until the objective function of the problem is optimized or the stop criterion is met.

CHAPTER 4

NUMERICAL EXAMPLES, RESULTS, AND DISCUSSIONS

4.1 Numerical Example

Numerical example for single item, we use data in Lee et al. [5] and for multi-item we create our own example based on approximation relative to Lee et al. [5].

Data demands for 20 periods of 3 different items (item 1, item 2, and item 3):

Table 1 Demands for 20 periods of 3 different items

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_{1t}	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
d_{2t}	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
d_{3t}	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67

Data ordering and transportation costs from 4 different suppliers (supplier A, supplier B, supplier C, and supplier D):

Table 2 Ordering costs, transportation cost per vehicle, and vehicle volume from 4 different suppliers

Company	A	B	C	D
Type of discount	All-unit	All-unit	Incremental	Incremental
Ordering cost	\$250	\$220	\$165	\$180
Transportation cost	\$21	\$22	\$19	\$21.50
Vehicle volume	9	9	9	9

Data holding and backordering cost:

Table 3 Item volume, holding cost and backordering cost

Item	1	2	3
Holding cost	$0.11	$0.09	$0.08
Shortage cost	$0.12	$0.10	$0.11
Item volume	0.36	0.40	0.43

Budget per replenishment: $ 30,000.00

Storage capacity: 3000 m^2 of items

Data quantity discount scheme from each supplier for each item.

Table 4 Quantity discount from supplier A for item 1

Min Q	Max Q	Price (P(Q))
0	350	$2.89
351	600	$2.75
601	~	$2.64

Table 5 Quantity discount from supplier B for item 1

Min Q	Max Q	Price (P(Q))
1	299	$2.90
300	750	$2.83
751	1000	$2.72
1001	~	$2.65

Table 6 Quantity discount from supplier C for item 1

Min Q	Max Q	Price (P(Q))
0	450	$2.95
451	800	$2.86
801	~	$2.73

Table 7 Quantity discount from supplier D for item 1

Min Q	Max Q	Price (P(Q))
1	450	\$2.88
451	600	\$2.72
601	900	\$2.69
901	~	\$2.66

Table 8 Quantity discount from supplier A for item 2

Min Q	Max Q	Price (P(Q))
0	250	\$1.89
251	500	\$1.84
501	~	\$1.78

Table 9 Quantity discount from supplier B for item 2

Min Q	Max Q	Price (P(Q))
1	299	\$1.89
300	600	\$1.83
601	899	\$1.74
900	~	\$1.70

Table 10 Quantity discount from supplier C for item 2

Min Q	Max Q	Price (P(Q))
0	250	\$1.91
251	750	\$1.87
751	~	\$1.84

Table 11 Quantity discount from supplier D for item 2

Min Q	Max Q	Price (P(Q))
1	400	\$1.89
401	600	\$1.82
601	900	\$1.74
901	~	\$1.70

Table 12 Quantity discount from supplier A for item 3

Min Q	Max Q	Price (P(Q))
0	350	\$3.40
351	600	\$3.37
601	~	\$3.30

Table 13 Quantity discount from supplier B for item 3

Min Q	Max Q	Price (P(Q))
1	299	\$3.42
300	750	\$3.38
751	1000	\$3.36
1001	~	\$3.30

Table 14 Quantity discount from supplier C for item 3

Min Q	Max Q	Price (P(Q))
0	450	\$3.40
451	800	\$3.33
801	~	\$3.25

Table 15 Quantity discount from supplier D for item 3

Min Q	Max Q	Price (P(Q))
1	450	\$3.40
451	600	\$3.38
601	900	\$3.36
901	~	\$3.33

For comparing the performance of both heuristic methods, for single item we use data from Lee et al. [5] and we generate random demand for 20 periods of replenishment for 10 cases. Here is the generated demand:

Table 16 Demands for 20 periods of 10 different cases (randomly generated)

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Case 1	561	1672	1053	2034	775	1110	1407	1962	1627	1424	455	1906	767	1420	1344	1447	973	411	1771	2016
Case 2	367	2074	2041	628	962	1116	1884	1568	2012	951	880	1465	577	472	1673	2055	779	1867	627	1874
Case 3	1524	1516	1695	631	1194	1549	545	648	1347	951	1586	439	742	321	850	1958	1179	1461	940	335
Case 4	1131	832	769	682	1494	498	1911	1773	1876	1532	1426	1490	639	1714	1599	1021	1869	1115	1250	1002
Case 5	1281	789	1189	599	2016	2045	818	1855	1973	1382	962	1709	1109	1974	1106	2022	2009	687	1115	1688
Case 6	934	348	934	367	1886	537	566	725	1504	567	994	922	464	1188	1655	404	1886	1617	1869	676
Case 7	1312	1071	1672	546	1233	1911	1418	1788	1254	1174	1919	1033	356	924	531	764	2035	1118	1382	839
Case 8	1443	880	1026	1772	2026	1510	475	1102	1163	1934	1534	1986	1448	1540	1233	1861	1969	976	1983	1107
Case 9	1365	1922	1155	401	1610	1223	1466	401	481	1062	891	691	1177	392	1638	1546	375	1414	941	1414
Case 10	1652	2071	2073	532	1660	1270	2070	1447	361	1920	2005	786	957	345	1939	1325	1866	1720	1720	1060

For comparing the performance of both heuristic methods, for multi item we generate random demand for 3 items for 20 periods of replenishment for 10 cases. Here is the generated demand:

Table 17 Demands for 3 items for 20 periods of 10 different cases (randomly generated)

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Case 1	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67
Case 2	534	540	384	379	413	109	260	417	418	272	544	509	508	492	282	364	460	148	106	149
	448	529	252	440	473	262	260	90	252	475	300	389	375	217	257	393	237	502	493	158
	472	307	86	297	180	540	148	282	161	207	519	485	228	361	250	146	489	486	209	119
Case 3	176	260	349	446	334	512	169	105	363	426	300	442	487	112	300	132	228	535	413	175
	486	195	293	179	370	330	220	155	426	334	493	510	99	243	518	265	401	429	198	295
	414	167	336	360	230	192	108	268	202	207	548	492	146	376	437	197	219	436	323	215
Case 4	443	294	294	191	280	320	94	204	477	418	532	222	130	270	471	429	401	234	370	440
	533	97	450	180	193	518	278	144	467	405	530	174	142	109	170	534	132	221	387	365
	130	344	321	439	487	222	138	128	239	395	339	418	449	299	374	270	335	356	257	380
Case 5	110	518	525	195	172	502	326	497	361	290	359	502	294	241	379	250	456	247	427	341
	170	550	537	234	503	206	210	518	409	129	326	542	295	382	473	512	146	149	525	260
	297	179	454	472	184	428	267	530	258	111	442	161	498	398	133	334	316	415	136	497
Case 6	445	193	225	368	467	377	223	326	470	115	354	321	205	369	184	443	286	134	380	440
	383	273	314	197	461	121	342	105	381	115	509	518	220	406	121	176	99	320	445	235
	198	158	203	186	512	254	439	169	313	211	242	481	395	465	205	346	454	409	203	354
Case 7	356	432	249	459	167	305	137	533	150	224	520	129	446	277	250	374	182	496	267	453
	521	454	228	185	125	335	306	288	324	98	401	101	202	383	213	112	131	115	343	200
	491	546	206	126	541	278	98	382	364	383	439	217	189	214	413	158	462	448	190	120
Case 8	118	239	433	305	370	542	212	357	268	376	216	372	221	118	489	125	213	411	369	201
	491	494	348	145	321	311	322	544	351	281	174	197	452	260	291	344	85	503	287	516
	107	463	416	133	88	486	480	322	193	454	330	526	486	389	193	368	194	300	550	310
Case 9	488	422	192	528	197	338	419	468	125	362	246	294	429	415	328	479	523	323	136	329
	438	339	380	453	111	191	168	182	454	87	396	314	517	182	521	407	208	532	85	133
	475	327	262	529	238	292	323	142	278	230	342	384	494	158	105	266	515	416	137	539
Case 10	541	389	535	454	131	210	492	328	407	146	311	341	137	540	133	445	86	533	484	368
	342	466	160	197	357	223	474	139	149	332	286	95	467	189	344	182	267	515	395	255
	516	137	119	186	335	158	420	492	146	268	322	448	232	505	211	309	438	96	195	86

4.2 Results and Comparisons

Here are the results for single-item problem for the case of 5 periods.

Table 18 MIP solution for scenario 1 obtained by LINGO

t	1	2	3	4	5
d_t	660	720	510	810	725
IB_t	0	0	2045	1535	725
IE_t	0	0	1535	725	0
SB_t	0	660	0	0	0
SE_t	660	1380	0	0	0
Q_t	0	0	3425	0	0

Objective value (total cost) =13,176.83, Computation time = 6 h

Table 19 MSM solution for scenario 1 obtained by MATLAB

t	1	2	3	4	5
d_t	660	720	510	810	725
IB_t	0	1230	510	1535	725
IE_t	0	510	0	725	0
SB_t	0	0	0	0	0
SE_t	660	0	0	0	0
Q_t	0	1890	0	1535	0

Objective value (total cost) =13,838.78, Computation time = 1 s

Table 20 GA solution for scenario 1 obtained by MATLAB

t	1	2	3	4	5
d_t	660	720	510	810	725
IB_t	0	0	2045	1535	725
IE_t	0	0	1535	725	0
SB_t	0	660	0	0	0
SE_t	660	1380	0	0	0
Q_t	0	0	3425	0	0

Objective value (total cost) =13,176.83, Computation time = 10 s

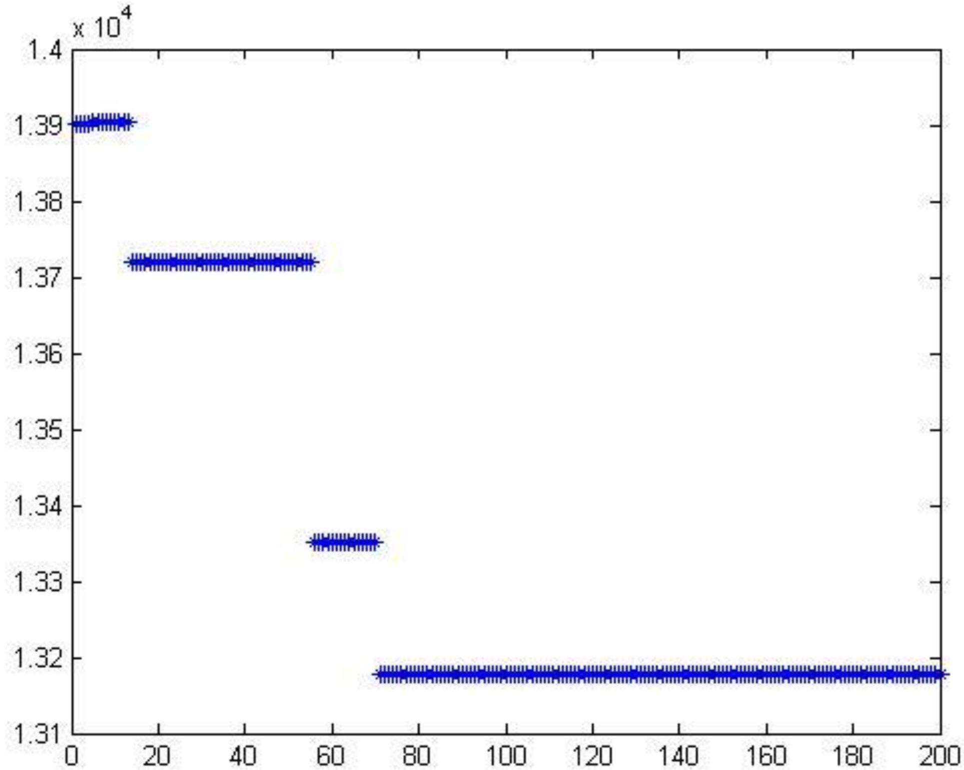

Figure 15 GA convergence for scenario 1

Here are the results for single-item problem for the case of 10 period.

Table 21 MIP solution for scenario 2 obtained by LINGO

t	1	2	3	4	5	6	7	8	9	10
d_t	660	720	510	810	725	465	1510	2410	515	1850
IB_t	0	1240	520	10	0	1975	1510	0	2365	1850
IE_t	0	520	10	0	0	1510	0	0	1850	0
SB_t	0	0	0	0	800	0	0	0	0	0
SE_t	660	0	0	800	1525	0	0	2410	0	0
Q_t	0	1900	0	0	0	3500	0	0	4775	0

Objective value (total cost) = 38,985.39, Computation time = 13 h

Table 22 MSM solution for scenario 2 obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10
d_t	660	720	510	810	725	465	1510	2410	515	1850
IB_t	0	1230	510	0	0	1975	1510	0	2365	1850
IE_t	0	510	0	0	0	1510	0	0	1850	0
SB_t	0	0	0	0	810	0	0	0	0	0
SE_t	660	0	0	810	1535	0	0	2410	0	0
Q_t	0	1890	0	0	0	3510	0	0	4775	0

Objective value (total cost) = 39,004.4, Computation time = 1 s

Table 23 GA solution for scenario 2 obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10
d_t	660	720	510	810	725	465	1510	2410	515	1850
IB_t	0	1230	510	0	1190	465	1510	2410	0	1850
IE_t	0	510	0	0	465	0	0	0	0	0
SB_t	0	0	0	0	0	0	0	0	0	0
SE_t	660	0	0	810	0	0	1150	0	515	0
Q_t	0	1890	0	0	2000	0	0	3920	0	2365

Objective value (total cost) = 39,013.5, Computation time = 23 s

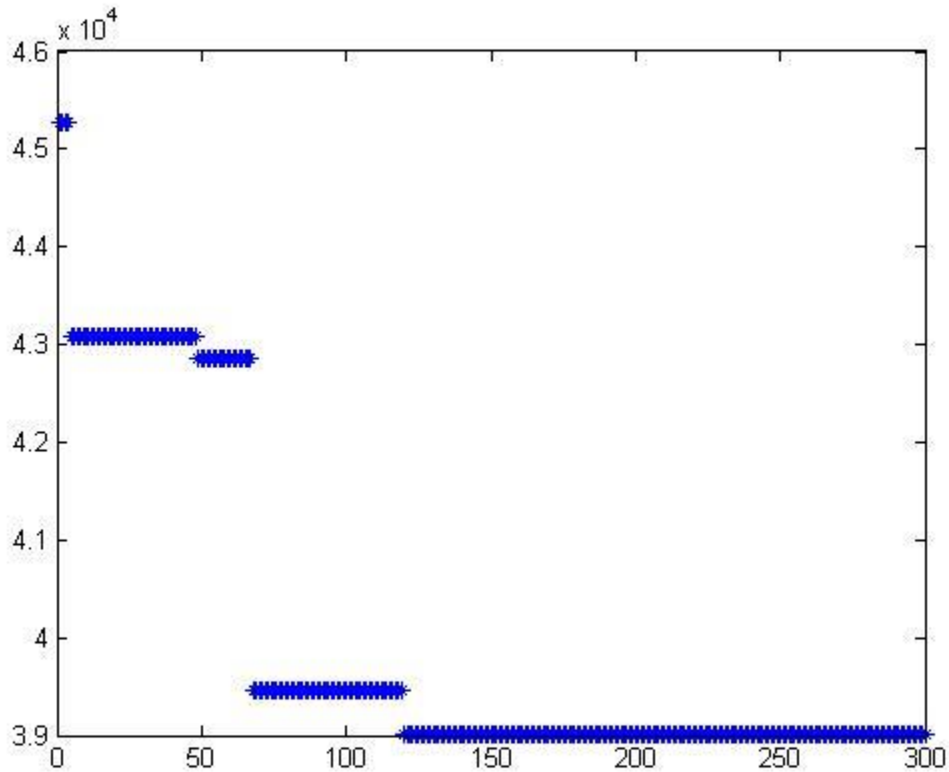

Figure 16 GA convergence for scenario 2

Table 24 MSM solution for scenario 3 obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	660	720	510	810	725	465	1510	2410	515	1850	320	540	590	250	780	830	950	650	410	195
IB_t	0	1230	510	0	1190	465	0	2925	515	0	1700	1380	840	250	0	0	2205	1255	605	195
IE_t	0	510	0	0	465	0	0	515	0	0	1380	840	250	0	0	0	1255	605	195	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	780	0	0	0	0
SE_t	660	0	0	810	0	0	1510	0	0	1850	0	0	0	0	780	1610	0	0	0	0
Q_t		1890			2000			4435			3550						3815			

Objective value (total cost) = 59,906.75, Calculation Time = 3 s

Table 25 GA solution for scenario 3 obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	660	720	510	810	725	465	1510	2410	515	1850	320	540	590	250	780	830	950	650	410	195
IB_t	0	0	2045	1535	725	0	3920	2410	0	3550	1700	1380	840	250	0	0	2205	1255	605	195
IE_t	0	0	1535	725	0	0	2410	0	0	1700	1380	840	250	0	0	0	1255	605	195	0
SB_t	0	660	0	0	0	0	0	0	0	0	0	0	0	0	0	780	0	0	0	0
SE_t	660	1380	0	0	0	465	0	0	515	0	0	0	0	0	780	1610	0	0	0	0
Q_t			3425				4385			4065							3815			

Objective value (total cost) = 59,756.05, Calculation Time = 80 s

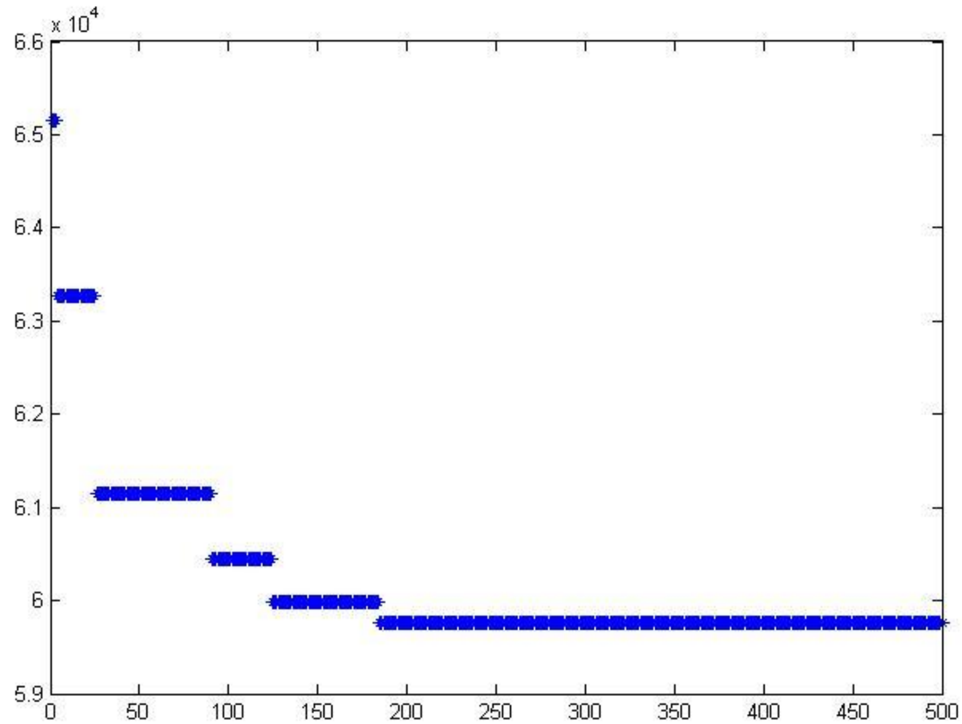

Figure 17 GA convergence for scenario 3

Here are the results for multi-item problem for the case of 4, 10, and 20 periods consecutively.

Table 26 MIP solution for scenario 1 multi items obtained by MATLAB

t	1	2	3	4
d_t	220	240	170	270
IB_t	0	0	440	270
IE_t	0	0	270	0
SB_t	0	220	0	0
SE_t	220	460	0	0
Q_t			900	
d_t	134	197	234	207
IB_t	0	0	441	207
IE_t	0	0	207	0
SB_t	0	134	0	0
SE_t	134	331	0	0
Q_t			772	
d_t	284	157	219	242
IB_t	0	0	461	242
IE_t	0	0	242	0
SB_t	0	284	0	0
SE_t	284	441	0	0
Q_t			902	

Objective value (total cost) = 9,640.38, Calculation Time = 96 h

Table 27 MSM solution for scenario 1 multi items obtained by MATLAB

t	1	2	3	4
d_t	220	240	170	270
IB_t	0	410	170	270
IE_t	0	170	0	0
SB_t	0	0	0	0
SE_t	220	0	0	0
Q_t		630		270
d_t	134	197	234	207
IB_t	0	431	234	207
IE_t	0	234	0	0
SB_t	0	0	0	0
SE_t	134	0	0	0
Q_t		565		207
d_t	284	157	219	242
IB_t	0	376	219	242
IE_t	0	219	0	0
SB_t	0	0	0	0
SE_t	284	0	0	0
Q_t		660		242

Objective value (total cost) = 9,790.20, Calculation Time = 2 s

Table 28 GA solution for scenario 1 multi items obtained by MATLAB

t	1	2	3	4
d_t	220	240	170	270
IB_t	0	0	440	270
IE_t	0	0	270	0
SB_t	0	220	0	0
SE_t	220	460	0	0
Q_t			900	
d_t	134	197	234	207
IB_t	0	0	441	207
IE_t	0	0	207	0
SB_t	0	134	0	0
SE_t	134	331	0	0
Q_t			772	
d_t	284	157	219	242
IB_t	0	0	461	242
IE_t	0	0	242	0
SB_t	0	284	0	0
SE_t	284	441	0	0
Q_t			902	

Objective value (total cost) = 9,640.38, Calculation Time = 22 s

Table 29 MSM solution for scenario 2 multi items obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10
d_t	220	240	170	270	242	155	504	804	172	617
IB_t	$0	410	170	0	397	155	0	976	172	617
IE_t	$0	170	0	0	155	0	0	172	0	0
SB_t	$0	0	0	0	0	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	0
Q_t		630			667			1480		617
d_t	134	197	234	207	179	300	240	214	142	202
IB_t	$0	431	234	0	479	300	0	356	142	202
IE_t	$0	234	0	0	300	0	0	142	0	0
SB_t	$0	0	0	0	0	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	0
Q_t		565			686			596		202
d_t	284	157	219	242	214	107	170	367	150	334
IB_t	0	376	219	0	321	107	0	517	150	334
IE_t	$0	219	0	0	107	0	0	150	0	0
SB_t	$0	0	0	0	0	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	0
Q_t		660			563			687		334

Objective value (total cost) = 28,699.43, Calculation Time = 3 s

Table 30 GA solution for scenario 2 multi items obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10
d_t	220	240	170	270	242	155	504	804	172	617
IB_t	0	410	170	0	397	155	0	1593	789	617
IE_t	0	170	0	0	155	0	0	789	617	0
SB_t	0	0	0	0	0	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	0
Q_t		630			667			2097		
d_t	134	197	234	207	179	300	240	214	142	202
IB_t	0	431	234	0	479	300	0	558	344	202
IE_t	0	234	0	0	300	0	0	344	202	0
SB_t	0	0	0	0	0	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	0
Q_t		565			686			798		
d_t	284	157	219	242	214	107	170	367	150	334
IB_t	0	376	219	0	321	107	0	851	484	668
IE_t	0	219	0	0	107	0	0	484	334	334
SB_t	0	0	0	0	0	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	0
Q_t		660			563			1021		334

Objective value (total cost) = 28,650.79, Calculation Time = 45 s

Table 31 MSM solution for scenario 3 multi items obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
IB_t	0	410	170	0	397	155	0	976	172	0	287	180	0	344	260	0	534	217	202	65
IE_t	0	170	0	0	155	0	0	172	0	0	180	0	0	260	0	0	217	0	65	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	617	0	0	197	0	0	277	0	0	0	0
Q_t		630			667			1480			904			541			811		202	
d_t	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
IB_t	0	431	234	0	479	300	0	356	142	0	487	187	0	464	174	0	410	260	518	184
IE_t	0	234	0	0	300	0	0	142	0	0	187	0	0	174	0	0	260	0	184	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	202	0	0	334	0	0	200	0	0	0	0
Q_t		565			686			596			689			798			610		518	
d_t	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67
IB_t	0	376	219	0	321	107	0	517	150	0	540	280	0	324	140	0	620	290	234	67
IE_t	0	219	0	0	107	0	0	150	0	0	280	0	0	140	0	0	290	0	67	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	334	0	0	250	0	0	300	0	0	0	0
Q_t		660			563			687			874			574			920		234	

Objective value (total cost) = 52,691.56, Calculation Time = 5 s

Table 32 GA solution for scenario 3 multi items obtained by MATLAB

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
IB_t	0	410	170	0	397	155	0	976	172	0	484	377	197	0	0	277	0	419	202	65
IE_t	0	170	0	0	155	0	0	172	0	0	377	197	0	0	0	0	0	202	65	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	84	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	617	0	0	0	84	344	0	317	0	0	0
Q_t		630			667			1480			1101					621		736		
d_t	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
IB_t	0	431	234	0	479	300	0	356	142	0	821	521	334	0	0	200	0	778	518	184
IE_t	0	234	0	0	300	0	0	142	0	0	521	334	0	0	0	0	0	518	184	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	290	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	202	0	0	0	290	464	0	150	0	0	0
Q_t		565			686			596			1023					664		928		
d_t	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67
IB_t	0	376	219	0	321	107	0	517	150	0	790	530	250	0	0	300	0	524	234	67
IE_t	0	219	0	0	107	0	0	150	0	0	530	250	0	0	0	0	0	234	67	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	184	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	334	0	0	0	184	324	0	330	0	0	0
Q_t		660			563			687			1124					624		854		

Objective value (total cost) = 52,603.60, Calculation Time = 70 s

For the lot-sizing example, the analytical MIP solution was obtained by LINGO 9.0, and the heuristic MSM solution was obtained by MATLAB. For scenario 1, which is for a 10-period planning horizon, solutions were obtained from both LINGO and MATLAB. Using LINGO 9.0., the MIP model produced the scenario 1 solution shown in Table 1, whose total cost is $38,985.39. Using MATLAB, the MSM heuristic produced a very similar scenario 1 solution. This MSM solution, shown in Table 2, has a total cost of $39,004.4. which is only 0.05% higher than the MIP solution. However, the calculation times of the MIP model and the MSM heuristic are extremely different. The MIP model needed 13 hours of computation, but the MSM took only 2 seconds to produce the solution. Therefore MSM method is quite effective for solving this problem.

Next, we attempted to solve scenario 2, which is for a 20-period planning horizon, using both the MIP and the MSM methods. For this scenario, however, LINGO was not able to solve the MIP problem. On the other hand, the MATLAB-implemented MSM method produced the solution shown in Table 3. The total cost is $60,442.3 and the computation time is only 3 seconds.

We compare the performance of both our heuristic method by applying the method in 10 different cases for both. We use data from Lee et al. [5] with some additional data. Demands for 20 periods of the 10 cases we generate it randomly. See all the results in the appendices.

Here are the summary of the results, we compare the objective value of both methods:

Table 33 Comparing the performance of MSM and GA for single-item based on the objective values

Objective Value	MSM Heuristic	GA Heuristic	Best Performance
Case 1	98,422.84	98,218.76	GA Heuristic
Case 2	97,662.99	97,164.29	GA Heuristic
Case 3	81,381.91	81,322.84	GA Heuristic
Case 4	96,638.63	96,908.03	MSM Heuristic
Case 5	106,498.33	106,791.24	MSM Heuristic
Case 6	76,096.94	76,096.94	-
Case 7	91,491.98	91,368.63	GA Heuristic
Case 8	108,578.63	108,969.97	MSM Heuristic
Case 9	82,064.58	81,387.76	GA Heuristic
Case 10	108,133.86	108,133.84	GA Heuristic

Table 34 Comparing the performance of MSM and GA for multi-item based on the objective values

Objective Value	MSM Heuristic	GA Heuristic	Best Performance
Case 1	52,691.56	52,603.60	GA Heuristic
Case 2	72,912.68	72,743.32	GA Heuristic
Case 3	68,092.12	67,873.19	GA Heuristic
Case 4	69,708.22	69,695.44	GA Heuristic
Case 5	75,097.09	74,994.31	GA Heuristic
Case 6	67,631.89	67,484.94	GA Heuristic
Case 7	66,260.67	66,143.87	GA Heuristic
Case 8	71,636.78	71,339.19	GA Heuristic
Case 9	72,256.82	72,209.61	GA Heuristic
Case 10	67,725.05	67,680.53	GA Heuristic

Table 35 Comparing the performance of both heuristic (MSM and GA) with the MIP model (optimal solution) based on the objective values

Objective Value	MSM Heuristic	GA Heuristic	MIP Solution	Best Heuristic	Percentage Error
Case 1 (5 periods)	23,287.73	22,846.49	22,846.49	GA Heuristic	0.00%
Case 2 (5 periods)	23,119.10	22,661.21	22,661.21	GA Heuristic	0.00%
Case 3 (5 periods)	24,911.44	24,638.37	24,638.37	GA Heuristic	0.00%
Case 4 (5 periods)	18,964.76	18,654.10	18,654.10	GA Heuristic	0.00%
Case 5 (5 periods)	22,530.68	22,243.01	22,243.01	GA Heuristic	0.00%
Case 6 (10 periods)	32,249.98	31,841.32	31,681.18	GA Heuristic	0.51%
Case 7 (10 periods)	50,416.45	50,293.17	49,930.16	GA Heuristic	0.73%
Case 8 (10 periods)	50,623.14	50,042.53	50,042.39	GA Heuristic	0.00%
Case 9 (10 periods)	41,968.46	41,855.67	41,834.75	GA Heuristic	0.05%
Case 10 (10 periods)	56,731.89	56,374.84	56,354.21	GA Heuristic	0.04%

4.3 Discussions

We develop model for multi-item lot sizing problem, therefore number of item will result on total number of variables use on the model. The model becomes more complex since we consider backordering. Hence we should find the way to simplify the model so that solver can solve the model within a reasonable computation time.

There are some options for simplifying the MIP model:

- Relaxing the objective function
- Linearizing the Objective function
- Reducing the number of variables
- Reducing the number of constraint

We chose the first two options for simplifying our MIP model. Lee et al. [5] developed a single item lot sizing problem with supplier selection using quantity discount, under no shortage. The model is obtained as MIP model, here is the objective function:

$$Minimize\, TC = \sum_{t=1}^{T}\left[\sum_{i=1}^{I}\left(o_i \times F_{it} + P\left(Q_{it}\right)\times \mathrm{Q}_{it} \times F_{it} + s_i \times \left\lceil \frac{Q_{it}}{b_i} \right\rceil\right) + \frac{h_j}{2}\times\left(2Y_{jt} - d_{jt}\right)\right]$$

The above objective function is in cubic form. The function $P\left(Q_{it}\right)\times \mathrm{Q}_{it} \times F_{it}$ is a cubic form since all $P\left(Q_{it}\right)$, Q_{it}, and F_{it} are unknown variables. If we follow this structure in our MIP model, the model become more complicated than Lee et al.'s model since our model is designed for multi-item, and also consider the shortage.

We also find a ceiling function on the objective function of Lee et al.'s model which is $\left\lceil \frac{Q_{it}}{b_i} \right\rceil$. This ceiling function may slower the computational process of the solver.

Therefore we need to relax model and try to linearize the model, at least for the objective function of the model.

The objective function of our model is minimizing the total cost including ordering cost, purchasing cost, transportation cost, holding cost and shortage cost. Ordering cost is in linear form. By removing the ceiling function, the transportation cost will be in linear form as well. Since we consider the selection of supplier based on their provided quantity discount, thus the purchasing cost is a cubic form which is $P\left(Q_{it}\right)\times \mathrm{Q}_{it} \times F_{it}$. Both holding cost and shortage cost are also non-linear forms.

For relaxing the objective function, we need to replace all those non-linear forms into linear form. We come up with the idea, instead of putting them on the objective function, we add them as constraints and replace each of them with a variable.

We end up with the following objective function:

$$Minimize\,TC = \sum_{t=1}^{T}\left(\sum_{j=1}^{J}\left(\sum_{i=1}^{I}\left(o_{ij} \times F_{ijt} + CP_{ijt}\right) + CI_{jt} + CS_{jt}\right) + \sum_{i=1}^{I} s_{it} \times N_{it}\right)$$

CHAPTER 5

CONCLUSIONS AND FUTURE

RECOMMENDATIONS

5.1 Conclusions

This thesis work presents the model of lot sizing problem for multi-item by considering supplier selection and given quantity discount from each supplier. The model was constructed under the assumption that demands are known for each item. Both inventory and shortage are considered in this model. At the beginning of each period there are three possibilities either there is inventory from the previous period, or there is a shortage, or both inventory and shortage level are zero. There is a trade of between holding for the inventory and backlogging for the shortage which is when the cost of backlogging smaller than holding cost, better to make a shortage of inventory rather than keep holding the inventory. The objective function of this model is the total cost for replenishment policy. This cost includes cost of ordering the items, transportation cost, holding cost, shortage cost, and purchase cost (depend on the quantity discount policy). Mixed integer programming has been formulated in order to solve the problem. Since the model uses many variables and constraints, many variables are binaries, the problem become more complicated and take a long time to solve using analytical method obtained by using LINGGO, the model become NP-Hard problem.

To tackle the NP-Hard problem we propose two heuristic methods. The first method is by modifying the Silver-Meal heuristic to solve lot sizing problem using quantity discount. The second model is by modeling the problem in a genetic algorithm. The results from both heuristic models show that both heuristic models are effective and efficient for solving lot sizing problem for multi-item with quantity discount.

5.2 Future Recommendations

This thesis work is only focus on lot sizing, therefore more complex supply chain problem can be considered for the future research. In this model we assumed that demand is known, however in real life mostly we find that demand is unknown, for the future research, stochastic demand can be considered. This model considered shortage cost for unmet demand, but there is no explanation where the cost comes from, for the future research, service level of the company can be considered as a way for calculating the shortage cost. In some cases the MSM heuristic give better results compare to GA heuristic, and therefore combining these two models will give better result or taking the MSM result as an initial solution.

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Appendices

Comparing the performance of MSM and GA for single-item based on the objective values

MSM solution for scenario 1

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	561	1672	1053	2034	775	1110	1407	1962	1627	1424	455	1906	767	1420	1344	1447	973	411	1771	2016
IB_t	0	2725	1053	0	3292	2517	1407	0	3051	1424	0	2673	767	0	2791	1447	0	0	3787	2016
IE_t	0	1053	0	0	2517	1407	0	0	1424	0	0	767	0	0	1447	0	0	0	2016	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	973	0	0
SE_t	561	0	0	2034	0	0	0	1962	0	0	455	0	0	1420	0	0	973	1384	0	0
Q_t		3286			5326				5013			3128			4211				5171	

Objective value (total cost) = 98,422.84

GA solution for scenario 1

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	561	1672	1053	2034	775	1110	1407	1962	1627	1424	455	1906	767	1420	1344	1447	973	411	1771	2016
IB_t	0	0	3087	2034	0	0	3369	1962	0	3785	2361	1906	0	4211	2791	1447	0	0	3787	2016
IE_t	0	0	2034	0	0	0	1962	0	0	2361	1906	0	0	2791	1447	0	0	0	2016	0
SB_t	0	561	0	0	0	775	0	0	0	0	0	0	0	0	0	0	0	973	0	0
SE_t	561	2233	0	0	775	1885	0	0	1627	0	0	0	767	0	0	0	973	1384	0	0
Q_t			5320				5254			5412				4978					5171	

Objective value (total cost) = 98,218.76

MSM solution for scenario 2

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	367	2074	2041	628	962	1116	1884	1568	2012	951	880	1465	577	472	1673	2055	779	1867	627	1874
IB_t	0	4115	2041	0	2078	1116	0	3580	2012	0	2345	1465	0	0	3728	2055	0	4368	2501	1874
IE_t	0	2041	0	0	1116	0	0	2012	0	0	1465	0	0	0	2055	0	0	2501	1874	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	577	0	0	0	0	0	0
SE_t	367	0	0	628	0	0	1884	0	0	951	0	0	577	1049	0	0	779	0	0	0
Q_t		4482			2706			5464			3296				4777			5147		

Objective value (total cost) = 97,662.99

GA solution for scenario 2

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	367	2074	2041	628	962	1116	1884	1568	2012	951	880	1465	577	472	1673	2055	779	1867	627	1874
IB_t	0	4115	2041	0	0	3000	1884	0	3843	1831	880	2514	1049	472	0	2834	779	0	2501	1874
IE_t	0	2041	0	0	0	1884	0	0	1831	880	0	1049	472	0	0	779	0	0	1874	0
SB_t	0	0	0	0	628	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	367	0	0	628	1590	0	0	1568	0	0	0	0	0	0	1673	0	0	1867	0	0
Q_t		4482				4590			5411			2514				4507			4368	

Objective value (total cost) = 97,164.29

MSM solution for scenario 3

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1524	1516	1695	631	1194	1549	545	648	1347	951	1586	439	742	321	850	1958	1179	1461	940	335
IB_t	0	3211	1695	0	3288	2094	545	0	2298	951	3088	1502	1063	321	0	3137	1179	2736	1275	335
IE_t	0	1695	0	0	2094	545	0	0	951	0	1502	1063	321	0	0	1179	0	1275	335	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1524	0	0	631	0	0	0	648	0	0	0	0	0	0	850	0	0	0	0	0
Q_t		4735			3919				2946		3088					3987		2736		

Objective value (total cost) = 81,381.91

GA solution for scenario 3

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1524	1516	1695	631	1194	1549	545	648	1347	951	1586	439	742	321	850	1958	1179	1461	940	335
IB_t	0	3842	2326	631	0	2742	1193	648	0	4889	3938	2352	1913	1171	850	0	3915	2736	1275	335
IE_t	0	2326	631	0	0	1193	648	0	0	3938	2352	1913	1171	850	0	0	2736	1275	335	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1524	0	0	0	1194	0	0	0	1347	0	0	0	0	0	0	1958	0	0	0	0
Q_t		5366				3936				6236							5873			

Objective value (total cost) = 81,322.84

MSM solution for scenario 4

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1131	832	769	682	1494	498	1911	1773	1876	1532	1426	1490	639	1714	1599	1021	1869	1115	1250	1002
IB_t	0	2283	1451	682	0	2409	1911	0	3408	1532	0	2129	639	0	4489	2890	1869	0	2252	1002
IE_t	0	1451	682	0	0	1911	0	0	1532	0	0	639	0	0	2890	1869	0	0	1002	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1131	0	0	0	1494	0	0	1773	0	0	1426	0	0	1714	0	0	0	1115	0	0
Q_t		3414				3903			5181			3555			6203				3367	

Objective value (total cost) = 96,638.63

GA solution for scenario 4

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1131	832	769	682	1494	498	1911	1773	1876	1532	1426	1490	639	1714	1599	1021	1869	1115	1250	1002
IB_t	0	0	3443	2674	1992	498	0	0	4834	2958	1426	0	3952	3313	1599	0	5236	3367	2252	1002
IE_t	0	0	2674	1992	498	0	0	0	2958	1426	0	0	3313	1599	0	0	3367	2252	1002	0
SB_t	0	1131	0	0	0	0	0	1911	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1131	1963	0	0	0	0	1911	3684	0	0	0	1490	0	0	0	1021	0	0	0	0
Q_t			5406						8518				5442				6257			

Objective value (total cost) = 96,908.03

MSM solution for scenario 5

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1281	789	1189	599	2016	2045	818	1855	1973	1382	962	1709	1109	1974	1106	2022	2009	687	1115	1688
IB_t	0	1978	1189	0	4879	2863	818	0	4317	2344	962	0	3083	1974	0	4031	2009	0	2803	1688
IE_t	0	1189	0	0	2863	818	0	0	2344	962	0	0	1974	0	0	2009	0	0	1688	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1281	0	0	599	0	0	0	1855	0	0	0	1709	0	0	1106	0	0	687	0	0
Q_t		3259			5478				6172				4792			5137			3490	

Objective value (total cost) = 106,498.33

GA solution for scenario 5

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1281	789	1189	599	2016	2045	818	1855	1973	1382	962	1709	1109	1974	1106	2022	2009	687	1115	1688
IB_t	0	2577	1788	599	0	4718	2673	1855	0	4053	2671	1709	0	5102	3128	2022	0	3490	2803	1688
IE_t	0	1788	599	0	0	2673	1855	0	0	2671	1709	0	0	3128	2022	0	0	2803	1688	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1281	0	0	0	2016	0	0	0	1973	0	0	0	1109	0	0	0	2009	0	0	0
Q_t		3858				6734				6026				6211				5499		

Objective value (total cost) = 106,791.24

MSM solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	934	348	934	367	1886	537	566	725	1504	567	994	922	464	1188	1655	404	1886	1617	1869	676
IB_t	0	1649	1301	367	0	1828	1291	725	0	2483	1916	922	0	3247	2059	404	0	4162	2545	676
IE_t	0	1301	367	0	0	1291	725	0	0	1916	922	0	0	2059	404	0	0	2545	676	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	934	0	0	0	1886	0	0	0	1504	0	0	0	464	0	0	0	1886	0	0	0
Q_t		2583				3714				3987				3711				6048		

Objective value (total cost) = 76,096.94

GA solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	934	348	934	367	1886	537	566	725	1504	567	994	922	464	1188	1655	404	1886	1617	1869	676
IB_t	0	1649	1301	367	0	1828	1291	725	0	2483	1916	922	0	3247	2059	404	0	4162	2545	676
IE_t	0	1301	367	0	0	1291	725	0	0	1916	922	0	0	2059	404	0	0	2545	676	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	934	0	0	0	1886	0	0	0	1504	0	0	0	464	0	0	0	1886	0	0	0
Q_t		2583				3714				3987				3711				6048		

Objective value (total cost) = 76,096.94

MSM solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1312	1071	1672	546	1233	1911	1418	1788	1254	1174	1919	1033	356	924	531	764	2035	1118	1382	839
IB_t	0	3289	2218	546	0	0	3206	1788	0	4126	2952	1033	0	2219	1295	764	0	3339	2221	839
IE_t	0	2218	546	0	0	0	1788	0	0	2952	1033	0	0	1295	764	0	0	2221	839	0
SB_t	0	0	0	0	0	1233	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1312	0	0	0	1233	3144	0	0	1254	0	0	0	356	0	0	0	2035	0	0	0
Q_t		4601					6350			5380				2575				5374		

Objective value (total cost) = 91,491.98

GA solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1312	1071	1672	546	1233	1911	1418	1788	1254	1174	1919	1033	356	924	531	764	2035	1118	1382	839
IB_t	0	3289	2218	546	0	0	4460	3042	1254	0	2952	1033	0	0	1295	764	0	3339	2221	839
IE_t	0	2218	546	0	0	0	3042	1254	0	0	1033	0	0	0	764	0	0	2221	839	0
SB_t	0	0	0	0	0	1233	0	0	0	0	0	0	0	356	0	0	0	0	0	0
SE_t	1312	0	0	0	1233	3144	0	0	0	1174	0	0	356	1280	0	0	2035	0	0	0
Q_t		4601					7604				4126				2575			5374		

Objective value (total cost) = 91,368.63

MSM solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1443	880	1026	1772	2026	1510	475	1102	1163	1934	1534	1986	1448	1540	1233	1861	1969	976	1983	1107
IB_t	0	1906	1026	0	4011	1985	475	0	3097	1934	0	3434	1448	0	3094	1861	0	4066	3090	1107
IE_t	0	1026	0	0	1985	475	0	0	1934	0	0	1448	0	0	1861	0	0	3090	1107	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1443	0	0	1772	0	0	0	1102	0	0	1534	0	0	1540	0	0	1969	0	0	0
Q_t		3349			5783				4199			4968			4634			6035		

Objective value (total cost) = 108,578.63

GA solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1443	880	1026	1772	2026	1510	475	1102	1163	1934	1534	1986	1448	1540	1233	1861	1969	976	1983	1107
IB_t	0	1906	1026	0	4011	1985	475	0	4631	3468	1534	0	4221	2773	1233	0	0	4066	3090	1107
IE_t	0	1026	0	0	1985	475	0	0	3468	1534	0	0	2773	1233	0	0	0	3090	1107	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1861	0	0	0
SE_t	1443	0	0	1772	0	0	0	1102	0	0	0	1986	0	0	0	1861	3830	0	0	0
Q_t		3349			5783				5733				6207					7896		

Objective value (total cost) = 108,969.97

MSM solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1365	1922	1155	401	1610	1223	1466	401	481	1062	891	691	1177	392	1638	1546	375	1414	941	1414
IB_t	0	3478	1556	401	0	3571	2348	882	481	0	1582	691	0	2030	1638	0	1789	1414	0	1414
IE_t	0	1556	401	0	0	2348	882	481	0	0	691	0	0	1638	0	0	1414	0	0	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1365	0	0	0	1610	0	0	0	0	1062	0	0	1177	0	0	1546	0	0	941	0
Q_t		4843				5181					2644			3207			3335			2355

Objective value (total cost) = 82,064.58

GA solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1365	1922	1155	401	1610	1223	1466	401	481	1062	891	691	1177	392	1638	1546	375	1414	941	1414
IB_t	0	3478	1556	401	0	3571	2348	882	481	0	1582	691	0	0	3559	1921	375	0	2355	1414
IE_t	0	1556	401	0	0	2348	882	481	0	0	691	0	0	0	1921	375	0	0	1414	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	1177	0	0	0	0	0	0
SE_t	1365	0	0	0	1610	0	0	0	0	1062	0	0	1177	1569	0	0	0	1414	0	0
Q_t		4843				5181					2644				5128				3769	

Objective value (total cost) = 81,387.76

MSM solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1652	2071	2073	532	1660	1270	2070	1447	361	1920	2005	786	957	345	1939	1325	1866	1720	1720	1060
IB_t	0	2071	0	2192	1660	0	3517	1447	0	0	2791	786	0	2284	1939	0	3586	1720	2780	1060
IE_t	0	0	0	1660	0	0	1447	0	0	0	786	0	0	1939	0	0	1720	0	1060	0
SB_t	0	0	0	0	0	0	0	0	0	361	0	0	0	0	0	0	0	0	0	0
SE_t	1652	0	2073	0	0	1270	0	0	361	2281	0	0	957	0	0	1325	0	0	0	0
Q_t		3723		4265			4787				5072			3241			4911		2780	

Objective value (total cost) = 108,133.86

GA solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	1652	2071	2073	532	1660	1270	2070	1447	361	1920	2005	786	957	345	1939	1325	1866	1720	1720	1060
IB_t	0	4144	2073	0	2930	1270	3878	1808	361	0	3748	1743	957	0	3264	1325	0	4500	2780	1060
IE_t	0	2073	0	0	1270	0	1808	361	0	0	1743	957	0	0	1325	0	0	2780	1060	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	1652	0	0	532	0	0	0	0	0	1920	0	0	0	345	0	0	1866	0	0	0
Q_t		5796			3462		3878				5668				3609			6366		

Objective value (total cost) = 108,133.84

Comparing the performance of MSM and GA for multi-item based on the objective values

MSM solution for scenario 1

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
IB_t	0	410	170	0	397	155	0	976	172	0	287	180	0	344	260	0	534	217	202	65
IE_t	0	170	0	0	155	0	0	172	0	0	180	0	0	260	0	0	217	0	65	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	617	0	0	197	0	0	277	0	0	0	0
Q_t		630			667			1480			904			541			811		202	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
IB_t	0	431	234	0	479	300	0	356	142	0	487	187	0	464	174	0	410	260	518	184
IE_t	0	234	0	0	300	0	0	142	0	0	187	0	0	174	0	0	260	0	184	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	202	0	0	334	0	0	200	0	0	0	0
Q_t		565			686			596			689			798			610		518	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67
IB_t	0	376	219	0	321	107	0	517	150	0	540	280	0	324	140	0	620	290	234	67
IE_t	0	219	0	0	107	0	0	150	0	0	280	0	0	140	0	0	290	0	67	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	334	0	0	250	0	0	300	0	0	0	0
Q_t		660			563			687			874			574			920		234	

Objective value (total cost) = 52,603.60

GA solution for scenario 1

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	220	240	170	270	242	155	504	804	172	617	107	180	197	84	260	277	317	217	137	65
IB_t	0	410	170	0	397	155	0	976	172	0	484	377	197	0	0	277	0	419	202	65
IE_t	0	170	0	0	155	0	0	172	0	0	377	197	0	0	0	0	0	202	65	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	84	0	0	0	0	0
SE_t	220	0	0	270	0	0	504	0	0	617	0	0	0	84	344	0	317	0	0	0
Q_t		630			667			1480			1101					621		736		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	134	197	234	207	179	300	240	214	142	202	300	187	334	290	174	200	150	260	334	184
IB_t	0	431	234	0	479	300	0	356	142	0	821	521	334	0	0	200	0	778	518	184
IE_t	0	234	0	0	300	0	0	142	0	0	521	334	0	0	0	0	0	518	184	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	290	0	0	0	0	0
SE_t	134	0	0	207	0	0	240	0	0	202	0	0	0	290	464	0	150	0	0	0
Q_t		565			686			596			1023					664		928		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	284	157	219	242	214	107	170	367	150	334	260	280	250	184	140	300	330	290	167	67
IB_t	0	376	219	0	321	107	0	517	150	0	790	530	250	0	0	300	0	524	234	67
IE_t	0	219	0	0	107	0	0	150	0	0	530	250	0	0	0	0	0	234	67	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	184	0	0	0	0	0
SE_t	284	0	0	242	0	0	170	0	0	334	0	0	0	184	324	0	330	0	0	0
Q_t		660			563			687			1124					624		854		

Objective value (total cost) = 52,603.60

MSM solution for scenario 2

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	534	540	384	379	413	109	260	417	418	272	544	509	508	492	282	364	460	148	106	149
IB_t	0	924	384	0	522	109	0	835	418	0	1053	509	0	774	282	0	608	148	255	149
IE_t	0	384	0	0	109	0	0	418	0	0	509	0	0	282	0	0	148	0	149	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	534	0	0	379	0	0	260	0	0	272	0	0	508	0	0	364	0	0	0	0
Q_t		1458			901			1095			1325			1282			972		255	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	448	529	252	440	473	262	260	90	252	475	300	389	375	217	257	393	237	502	493	158
IB_t	0	781	252	0	735	262	0	342	252	0	689	389	0	474	257	0	739	502	651	158
IE_t	0	252	0	0	262	0	0	252	0	0	389	0	0	257	0	0	502	0	158	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	448	0	0	440	0	0	260	0	0	475	0	0	375	0	0	393	0	0	0	0
Q_t		1229			1175			602			1164			849			1132		651	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	472	307	86	297	180	540	148	282	161	207	519	485	228	361	250	146	489	486	209	119
IB_t	0	393	86	0	720	540	0	443	161	0	1004	485	0	611	250	0	975	486	328	119
IE_t	0	86	0	0	540	0	0	161	0	0	485	0	0	250	0	0	486	0	119	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	472	0	0	297	0	0	148	0	0	207	0	0	228	0	0	146	0	0	0	0
Q_t		865			1017			591			1211			839			1121		328	

Objective value (total cost) = 72,912.68

GA solution for scenario 2

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	534	540	384	379	413	109	260	417	418	272	544	509	508	492	282	364	460	148	106	149
IB_t	0	924	384	0	782	369	260	0	690	272	0	1017	508	0	646	364	0	403	255	149
IE_t	0	384	0	0	369	260	0	0	272	0	0	508	0	0	364	0	0	255	149	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	534	0	0	379	0	0	0	417	0	0	544	0	0	492	0	0	460	0	0	0
Q_t		1458			1161				1107			1561			1138			863		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	448	529	252	440	473	262	260	90	252	475	300	389	375	217	257	393	237	502	493	158
IB_t	0	781	252	0	995	522	260	0	727	475	0	764	375	0	650	393	0	1153	651	158
IE_t	0	252	0	0	522	260	0	0	475	0	0	375	0	0	393	0	0	651	158	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	448	0	0	440	0	0	0	90	0	0	300	0	0	217	0	0	237	0	0	0
Q_t		1229			1435				817			1064			867			1390		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	472	307	86	297	180	540	148	282	161	207	519	485	228	361	250	146	489	486	209	119
IB_t	0	393	86	0	868	688	148	0	368	207	0	713	228	0	396	146	0	814	328	119
IE_t	0	86	0	0	688	148	0	0	207	0	0	228	0	0	146	0	0	328	119	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	472	0	0	297	0	0	0	282	0	0	519	0	0	361	0	0	489	0	0	0
Q_t		865			1165				650			1232			757			1303		

Objective value (total cost) = 72,743.32

MSM solution for scenario 3

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	176	260	349	446	334	512	169	105	363	426	300	442	487	112	300	132	228	535	413	175
IB_t	0	609	349	0	846	512	0	468	363	0	742	442	0	412	300	0	763	535	588	175
IE_t	0	349	0	0	512	0	0	363	0	0	442	0	0	300	0	0	535	0	175	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	176	0	0	446	0	0	169	0	0	426	0	0	487	0	0	132	0	0	0	0
Q_t		785			1292			637			1168			899			895		588	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	486	195	293	179	370	330	220	155	426	334	493	510	99	243	518	265	401	429	198	295
IB_t	0	488	293	0	700	330	0	581	426	0	1003	510	0	761	518	0	830	429	493	295
IE_t	0	293	0	0	330	0	0	426	0	0	510	0	0	518	0	0	429	0	295	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	486	0	0	179	0	0	220	0	0	334	0	0	99	0	0	265	0	0	0	0
Q_t		974			879			801			1337			860			1095		493	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	414	167	336	360	230	192	108	268	202	207	548	492	146	376	437	197	219	436	323	215
IB_t	0	503	336	0	422	192	0	470	202	0	1040	492	0	813	437	0	655	436	538	215
IE_t	0	336	0	0	192	0	0	202	0	0	492	0	0	437	0	0	436	0	215	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	414	0	0	360	0	0	108	0	0	207	0	0	146	0	0	197	0	0	0	0
Q_t		917			782			578			1247			959			852		538	

Objective value (total cost) = 68,092.12

GA solution for scenario 3

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	176	260	349	446	334	512	169	105	363	426	300	442	487	112	300	132	228	535	413	175
IB_t	0	609	349	0	1120	786	274	105	0	726	300	0	599	112	432	132	0	1123	588	175
IE_t	0	349	0	0	786	274	105	0	0	300	0	0	112	0	132	0	0	588	175	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	176	0	0	446	0	0	0	0	363	0	0	442	0	0	0	0	228	0	0	0
Q_t		785			1566					1089			1041		432			1351		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	486	195	293	179	370	330	220	155	426	334	493	510	99	243	518	265	401	429	198	295
IB_t	0	488	293	0	1075	705	375	155	0	827	493	0	342	243	783	265	0	922	493	295
IE_t	0	293	0	0	705	375	155	0	0	493	0	0	243	0	265	0	0	493	295	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	486	0	0	179	0	0	0	0	426	0	0	510	0	0	0	0	401	0	0	0
Q_t		974			1254					1253			852		783			1323		

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	414	167	336	360	230	192	108	268	202	207	548	492	146	376	437	197	219	436	323	215
IB_t	0	503	336	0	798	568	376	268	0	755	548	0	522	376	634	197	0	974	538	215
IE_t	0	336	0	0	568	376	268	0	0	548	0	0	376	0	197	0	0	538	215	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	414	0	0	360	0	0	0	0	202	0	0	492	0	0	0	0	219	0	0	0
Q_t		917			1158					957			1014		634			1193		

Objective value (total cost) = 67,873.19

MSM solution for scenario 4

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	443	294	294	191	280	320	94	204	477	418	532	222	130	270	471	429	401	234	370	440
IB_t	0	588	294	0	600	320	0	681	477	0	754	222	0	741	471	0	635	234	810	440
IE_t	0	294	0	0	320	0	0	477	0	0	222	0	0	471	0	0	234	0	440	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	443	0	0	191	0	0	94	0	0	418	0	0	130	0	0	429	0	0	0	0
Q_t		1031			791			775			1172			871			1064		810	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	533	97	450	180	193	518	278	144	467	405	530	174	142	109	170	534	132	221	387	365
IB_t	0	547	450	0	711	518	0	611	467	0	704	174	0	279	170	0	353	221	752	365
IE_t	0	450	0	0	518	0	0	467	0	0	174	0	0	170	0	0	221	0	365	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	533	0	0	180	0	0	278	0	0	405	0	0	142	0	0	534	0	0	0	0
Q_t		1080			891			889			1109			421			887		752	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	130	344	321	439	487	222	138	128	239	395	339	418	449	299	374	270	335	356	257	380
IB_t	0	665	321	0	709	222	0	367	239	0	757	418	0	673	374	0	691	356	637	380
IE_t	0	321	0	0	222	0	0	239	0	0	418	0	0	374	0	0	356	0	380	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	130	0	0	439	0	0	138	0	0	395	0	0	449	0	0	270	0	0	0	0
Q_t		795			1148			505			1152			1122			961		637	

Objective value (total cost) = 69,708.22

GA solution for scenario 4

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	443	294	294	191	280	320	94	204	477	418	532	222	130	270	471	429	401	234	370	440
IB_t	0	0	765	471	280	0	298	204	0	1572	1154	622	400	270	0	0	1445	1044	810	440
IE_t	0	0	471	280	0	0	204	0	0	1154	622	400	270	0	0	0	1044	810	440	0
SB_t	0	443	0	0	0	0	0	0	0	0	0	0	0	0	0	471	0	0	0	0
SE_t	443	737	0	0	0	320	0	0	477	0	0	0	0	0	471	900	0	0	0	0
Q_t			1502				618			2049							2345			

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	533	97	450	180	193	518	278	144	467	405	530	174	142	109	170	534	132	221	387	365
IB_t	0	0	823	373	193	0	422	144	0	1360	955	425	251	109	0	0	1105	973	752	365
IE_t	0	0	373	193	0	0	144	0	0	955	425	251	109	0	0	0	973	752	365	0
SB_t	0	533	0	0	0	0	0	0	0	0	0	0	0	0	0	170	0	0	0	0
SE_t	533	630	0	0	0	518	0	0	467	0	0	0	0	0	170	704	0	0	0	0
Q_t			1453				940			1827							1809			

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	130	344	321	439	487	222	138	128	239	395	339	418	449	299	374	270	335	356	257	380
IB_t	0	0	1247	926	487	0	266	128	0	1900	1505	1166	748	299	0	0	1328	993	637	380
IE_t	0	0	926	487	0	0	128	0	0	1505	1166	748	299	0	0	0	993	637	380	0
SB_t	0	130	0	0	0	0	0	0	0	0	0	0	0	0	0	374	0	0	0	0
SE_t	130	474	0	0	0	222	0	0	239	0	0	0	0	0	374	644	0	0	0	0
Q_t			1721				488			2139							1972			

Objective value (total cost) = 69,695.44

MSM solution for scenario 5

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	110	518	525	195	172	502	326	497	361	290	359	502	294	241	379	250	456	247	427	341
IB_t	0	1043	525	0	674	502	0	858	361	0	861	502	0	620	379	0	703	247	768	341
IE_t	0	525	0	0	502	0	0	361	0	0	502	0	0	379	0	0	247	0	341	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	110	0	0	195	0	0	326	0	0	290	0	0	294	0	0	250	0	0	0	0
Q_t		1153			869			1184			1151			914			953		768	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	170	550	537	234	503	206	210	518	409	129	326	542	295	382	473	512	146	149	525	260
IB_t	0	1087	537	0	709	206	0	927	409	0	868	542	0	855	473	0	295	149	785	260
IE_t	0	537	0	0	206	0	0	409	0	0	542	0	0	473	0	0	149	0	260	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	170	0	0	234	0	0	210	0	0	129	0	0	295	0	0	512	0	0	0	0
Q_t		1257			943			1137			997			1150			807		785	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	297	179	454	472	184	428	267	530	258	111	442	161	498	398	133	334	316	415	136	497
IB_t	0	633	454	0	612	428	0	788	258	0	603	161	0	531	133	0	731	415	633	497
IE_t	0	454	0	0	428	0	0	258	0	0	161	0	0	133	0	0	415	0	497	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	297	0	0	472	0	0	267	0	0	111	0	0	498	0	0	334	0	0	0	0
Q_t		930			1084			1055			714			1029			1065		633	

Objective value (total cost) = 75,097.09

GA solution for scenario 5

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	110	518	525	195	172	502	326	497	361	290	359	502	294	241	379	250	456	247	427	341
IB_t	0	1043	525	0	674	502	0	1148	651	290	0	1037	535	241	0	706	456	0	768	341
IE_t	0	525	0	0	502	0	0	651	290	0	0	535	241	0	0	456	0	0	341	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	110	0	0	195	0	0	326	0	0	0	359	0	0	0	379	0	0	247	0	0
Q_t		1153			869			1474				1396				1085			1015	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	170	550	537	234	503	206	210	518	409	129	326	542	295	382	473	512	146	149	525	260
IB_t	0	1087	537	0	709	206	0	1056	538	129	0	1219	677	382	0	658	146	0	785	260
IE_t	0	537	0	0	206	0	0	538	129	0	0	677	382	0	0	146	0	0	260	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	170	0	0	234	0	0	210	0	0	0	326	0	0	0	473	0	0	149	0	0
Q_t		1257			943			1266				1545				1131			934	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	297	179	454	472	184	428	267	530	258	111	442	161	498	398	133	334	316	415	136	497
IB_t	0	633	454	0	612	428	0	899	369	111	0	1057	896	398	0	650	316	0	633	497
IE_t	0	454	0	0	428	0	0	369	111	0	0	896	398	0	0	316	0	0	497	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	297	0	0	472	0	0	267	0	0	0	442	0	0	0	133	0	0	415	0	0
Q_t		930			1084			1166				1499				783			1048	

Objective value (total cost) = 74,994.31

MSM solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	445	193	225	368	467	377	223	326	470	115	354	321	205	369	184	443	286	134	380	440
IB_t	0	418	225	0	844	377	0	796	470	0	675	321	0	553	184	0	420	134	820	440
IE_t	0	225	0	0	377	0	0	470	0	0	321	0	0	184	0	0	134	0	440	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	445	0	0	368	0	0	223	0	0	115	0	0	205	0	0	443	0	0	0	0
Q_t		863			1212			1019			790			758			863		820	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	383	273	314	197	461	121	342	105	381	115	509	518	220	406	121	176	99	320	445	235
IB_t	0	587	314	0	582	121	0	486	381	0	1027	518	0	527	121	0	419	320	680	235
IE_t	0	314	0	0	121	0	0	381	0	0	518	0	0	121	0	0	320	0	235	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	383	0	0	197	0	0	342	0	0	115	0	0	220	0	0	176	0	0	0	0
Q_t		970			779			828			1142			747			595		680	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	198	158	203	186	512	254	439	169	313	211	242	481	395	465	205	346	454	409	203	354
IB_t	0	361	203	0	766	254	0	482	313	0	723	481	0	670	205	0	863	409	557	354
IE_t	0	203	0	0	254	0	0	313	0	0	481	0	0	205	0	0	409	0	354	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	198	0	0	186	0	0	439	0	0	211	0	0	395	0	0	346	0	0	0	0
Q_t		559			952			921			934			1065			1209		557	

Objective value (total cost) = 67,631.89

GA solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	445	193	225	368	467	377	223	326	470	115	354	321	205	369	184	443	286	134	380	440
IB_t	0	418	225	0	844	377	0	911	585	115	0	526	205	0	913	729	286	0	820	440
IE_t	0	225	0	0	377	0	0	585	115	0	0	205	0	0	729	286	0	0	440	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	445	0	0	368	0	0	223	0	0	0	354	0	0	369	0	0	0	134	0	0
Q_t		863			1212			1134				880			1282				954	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	383	273	314	197	461	121	342	105	381	115	509	518	220	406	121	176	99	320	445	235
IB_t	0	587	314	0	582	121	0	601	496	115	0	738	220	0	396	275	99	0	680	235
IE_t	0	314	0	0	121	0	0	496	115	0	0	220	0	0	275	99	0	0	235	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	383	0	0	197	0	0	342	0	0	0	509	0	0	406	0	0	0	320	0	0
Q_t		970			779			943				1247			802				1000	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	198	158	203	186	512	254	439	169	313	211	242	481	395	465	205	346	454	409	203	354
IB_t	0	361	203	0	766	254	0	693	524	211	0	876	395	0	1005	800	454	0	557	354
IE_t	0	203	0	0	254	0	0	524	211	0	0	395	0	0	800	454	0	0	354	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	198	0	0	186	0	0	439	0	0	0	242	0	0	465	0	0	0	409	0	0
Q_t		559			952			1132				1118			1470				966	

Objective value (total cost) = 67,484.94

MSM solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	356	432	249	459	167	305	137	533	150	224	520	129	446	277	250	374	182	496	267	453
IB_t	0	681	249	0	472	305	0	683	150	0	649	129	0	527	250	0	678	496	720	453
IE_t	0	249	0	0	305	0	0	150	0	0	129	0	0	250	0	0	496	0	453	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	356	0	0	459	0	0	137	0	0	224	0	0	446	0	0	374	0	0	0	0
Q_t		1037			931			820			873			973			1052		720	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	521	454	228	185	125	335	306	288	324	98	401	101	202	383	213	112	131	115	343	200
IB_t	0	682	228	0	460	335	0	612	324	0	502	101	0	596	213	0	246	115	543	200
IE_t	0	228	0	0	335	0	0	324	0	0	101	0	0	213	0	0	115	0	200	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	521	0	0	185	0	0	306	0	0	98	0	0	202	0	0	112	0	0	0	0
Q_t		1203			645			918			600			798			358		543	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	491	546	206	126	541	278	98	382	364	383	439	217	189	214	413	158	462	448	190	120
IB_t	0	752	206	0	819	278	0	746	364	0	656	217	0	627	413	0	910	448	310	120
IE_t	0	206	0	0	278	0	0	364	0	0	217	0	0	413	0	0	448	0	120	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	491	0	0	126	0	0	98	0	0	383	0	0	189	0	0	158	0	0	0	0
Q_t		1243			945			844			1039			816			1068		310	

Objective value (total cost) = 66,260.67

GA solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	356	432	249	459	167	305	137	533	150	224	520	129	446	277	250	374	182	496	267	453
IB_t	0	681	249	0	609	442	137	0	374	224	0	852	723	277	0	556	182	0	720	453
IE_t	0	249	0	0	442	137	0	0	224	0	0	723	277	0	0	182	0	0	453	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	356	0	0	459	0	0	0	533	0	0	520	0	0	0	250	0	0	496	0	0
Q_t		1037			1068				907			1372				806			1216	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	521	454	228	185	125	335	306	288	324	98	401	101	202	383	213	112	131	115	343	200
IB_t	0	682	228	0	766	641	306	0	422	98	0	686	585	383	0	243	131	0	543	200
IE_t	0	228	0	0	641	306	0	0	98	0	0	585	383	0	0	131	0	0	200	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	521	0	0	185	0	0	0	288	0	0	401	0	0	0	213	0	0	115	0	0
Q_t		1203			951				710			1087				456			658	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	491	546	206	126	541	278	98	382	364	383	439	217	189	214	413	158	462	448	190	120
IB_t	0	752	206	0	917	376	98	0	747	383	0	620	403	214	0	620	462	0	310	120
IE_t	0	206	0	0	376	98	0	0	383	0	0	403	214	0	0	462	0	0	120	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	491	0	0	126	0	0	0	382	0	0	439	0	0	0	413	0	0	448	0	0
Q_t		1243			1043				1129			1059				1033			758	

Objective value (total cost) = 66,143.87

MSM solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	118	239	433	305	370	542	212	357	268	376	216	372	221	118	489	125	213	411	369	201
IB_t	0	672	433	0	912	542	0	625	268	0	588	372	0	607	489	0	624	411	570	201
IE_t	0	433	0	0	542	0	0	268	0	0	372	0	0	489	0	0	411	0	201	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	118	0	0	305	0	0	212	0	0	376	0	0	221	0	0	125	0	0	0	0
Q_t		790			1217			837			964			828			749		570	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	491	494	348	145	321	311	322	544	351	281	174	197	452	260	291	344	85	503	287	516
IB_t	0	842	348	0	632	311	0	895	351	0	371	197	0	551	291	0	588	503	803	516
IE_t	0	348	0	0	311	0	0	351	0	0	197	0	0	291	0	0	503	0	516	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	491	0	0	145	0	0	322	0	0	281	0	0	452	0	0	344	0	0	0	0
Q_t		1333			777			1217			652			1003			932		803	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	107	463	416	133	88	486	480	322	193	454	330	526	486	389	193	368	194	300	550	310
IB_t	0	879	416	0	574	486	0	515	193	0	856	526	0	582	193	0	494	300	860	310
IE_t	0	416	0	0	486	0	0	193	0	0	526	0	0	193	0	0	300	0	310	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	107	0	0	133	0	0	480	0	0	454	0	0	486	0	0	368	0	0	0	0
Q_t		986			707			995			1310			1068			862		860	

Objective value (total cost) = 71,636.78

GA solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	118	239	433	305	370	542	212	357	268	376	216	372	221	118	489	125	213	411	369	201
IB_t	0	977	738	305	0	754	212	0	644	376	0	711	339	118	0	338	213	0	570	201
IE_t	0	738	305	0	0	212	0	0	376	0	0	339	118	0	0	213	0	0	201	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	118	0	0	0	370	0	0	357	0	0	216	0	0	0	489	0	0	411	0	0
Q_t		1095				1124			1001			927				827			981	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	491	494	348	145	321	311	322	544	351	281	174	197	452	260	291	344	85	503	287	516
IB_t	0	987	493	145	0	633	322	0	632	281	0	909	712	260	0	429	85	0	803	516
IE_t	0	493	145	0	0	322	0	0	281	0	0	712	260	0	0	85	0	0	516	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	491	0	0	0	321	0	0	544	0	0	174	0	0	0	291	0	0	503	0	0
Q_t		1478				954			1176			1083				720			1306	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	107	463	416	133	88	486	480	322	193	454	330	526	486	389	193	368	194	300	550	310
IB_t	0	1012	549	133	0	966	480	0	647	454	0	1401	875	389	0	562	194	0	860	310
IE_t	0	549	133	0	0	480	0	0	454	0	0	875	389	0	0	194	0	0	310	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	107	0	0	0	88	0	0	322	0	0	330	0	0	0	193	0	0	300	0	0
Q_t		1119				1054			969			1731				755			1160	

Objective value (total cost) = 71,339.19

MSM solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	488	422	192	528	197	338	419	468	125	362	246	294	429	415	328	479	523	323	136	329
IB_t	0	614	192	0	535	338	0	593	125	0	540	294	0	743	328	0	846	323	465	329
IE_t	0	192	0	0	338	0	0	125	0	0	294	0	0	328	0	0	323	0	329	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	488	0	0	528	0	0	419	0	0	362	0	0	429	0	0	479	0	0	0	0
Q_t		1102			1063			1012			902			1172			1325		465	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	438	339	380	453	111	191	168	182	454	87	396	314	517	182	521	407	208	532	85	133
IB_t	0	719	380	0	302	191	0	636	454	0	710	314	0	703	521	0	740	532	218	133
IE_t	0	380	0	0	191	0	0	454	0	0	314	0	0	521	0	0	532	0	133	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	438	0	0	453	0	0	168	0	0	87	0	0	517	0	0	407	0	0	0	0
Q_t		1157			755			804			797			1220			1147		218	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	475	327	262	529	238	292	323	142	278	230	342	384	494	158	105	266	515	416	137	539
IB_t	0	589	262	0	530	292	0	420	278	0	726	384	0	263	105	0	931	416	676	539
IE_t	0	262	0	0	292	0	0	278	0	0	384	0	0	105	0	0	416	0	539	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	475	0	0	529	0	0	323	0	0	230	0	0	494	0	0	266	0	0	0	0
Q_t		1064			1059			743			956			757			1197		676	

Objective value (total cost) = 72,256.82

GA solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	488	422	192	528	197	338	419	468	125	362	246	294	429	415	328	479	523	323	136	329
IB_t	910	422	0	1063	535	338	0	955	487	362	0	0	844	415	0	1002	523	0	465	329
IE_t	422	0	0	535	338	0	0	487	362	0	0	0	415	0	0	523	0	0	329	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	246	0	0	0	0	0	0	0	0
SE_t	0	0	192	0	0	0	419	0	0	0	246	540	0	0	328	0	0	323	0	0
Q_t	910			1255				1374					1384			1330			788	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	438	339	380	453	111	191	168	182	454	87	396	314	517	182	521	407	208	532	85	133
IB_t	777	339	0	755	302	191	0	723	541	87	0	0	699	182	0	615	208	0	218	133
IE_t	339	0	0	302	191	0	0	541	87	0	0	0	182	0	0	208	0	0	133	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	396	0	0	0	0	0	0	0	0
SE_t	0	0	380	0	0	0	168	0	0	0	396	710	0	0	521	0	0	532	0	0
Q_t	777			1135				891					1409			1136			750	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	475	327	262	529	238	292	323	142	278	230	342	384	494	158	105	266	515	416	137	539
IB_t	802	327	0	1059	530	292	0	650	508	230	0	0	652	158	0	781	515	0	676	539
IE_t	327	0	0	530	292	0	0	508	230	0	0	0	158	0	0	515	0	0	539	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	342	0	0	0	0	0	0	0	0
SE_t	0	0	262	0	0	0	323	0	0	0	342	726	0	0	105	0	0	416	0	0
Q_t	802			1321				973					1378			886			1092	

Objective value (total cost) = 72,209.61

MSM solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	541	389	535	454	131	210	492	328	407	146	311	341	137	540	133	445	86	533	484	368
IB_t	0	924	535	0	341	210	0	735	407	0	652	341	0	673	133	0	619	533	852	368
IE_t	0	535	0	0	210	0	0	407	0	0	341	0	0	133	0	0	533	0	368	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	541	0	0	454	0	0	492	0	0	146	0	0	137	0	0	445	0	0	0	0
Q_t		1465			795			1227			798			810			1064		852	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	342	466	160	197	357	223	474	139	149	332	286	95	467	189	344	182	267	515	395	255
IB_t	0	626	160	0	580	223	0	288	149	0	381	95	0	533	344	0	782	515	650	255
IE_t	0	160	0	0	223	0	0	149	0	0	95	0	0	344	0	0	515	0	255	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	342	0	0	197	0	0	474	0	0	332	0	0	467	0	0	182	0	0	0	0
Q_t		968			777			762			713			1000			964		650	
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	516	137	119	186	335	158	420	492	146	268	322	448	232	505	211	309	438	96	195	86
IB_t	0	256	119	0	493	158	0	638	146	0	770	448	0	716	211	0	534	96	281	86
IE_t	0	119	0	0	158	0	0	146	0	0	448	0	0	211	0	0	96	0	86	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
SE_t	516	0	0	186	0	0	420	0	0	268	0	0	232	0	0	309	0	0	0	0
Q_t		772			679			1058			1038			948			843		281	

Objective value (total cost) = 67,725.05

GA solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	541	389	535	454	131	210	492	328	407	146	311	341	137	540	133	445	86	533	484	368
IB_t	930	389	0	795	341	210	0	881	553	146	0	0	677	540	0	531	86	0	852	368
IE_t	389	0	0	341	210	0	0	553	146	0	0	0	540	0	0	86	0	0	368	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	311	0	0	0	0	0	0	0	0
SE_t	0	0	535	0	0	0	492	0	0	0	311	652	0	0	133	0	0	533	0	0
Q_t	930			1330				1373					1329			664			1385	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	342	466	160	197	357	223	474	139	149	332	286	95	467	189	344	182	267	515	395	255
IB_t	808	466	0	777	580	223	0	620	481	332	0	0	656	189	0	449	267	0	650	255
IE_t	466	0	0	580	223	0	0	481	332	0	0	0	189	0	0	267	0	0	255	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	286	0	0	0	0	0	0	0	0
SE_t	0	0	160	0	0	0	474	0	0	0	286	381	0	0	344	0	0	515	0	0
Q_t	808			937				1094					1037			793			1165	

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d_t	516	137	119	186	335	158	420	492	146	268	322	448	232	505	211	309	438	96	195	86
IB_t	653	137	0	679	493	158	0	906	414	268	0	0	737	505	0	747	438	0	281	86
IE_t	137	0	0	493	158	0	0	414	268	0	0	0	505	0	0	438	0	0	86	0
SB_t	0	0	0	0	0	0	0	0	0	0	0	322	0	0	0	0	0	0	0	0
SE_t	0	0	119	0	0	0	420	0	0	0	322	770	0	0	211	0	0	96	0	0
Q_t	653			798				1326					1507			958			377	

Objective value (total cost) = 67,680.53

Comparing the performance of heuristic both model (MSM and GA) with MIP model based on the objective values

MIP solution for scenario 1

t	1	2	3	4	5
d_t	561	1672	1053	2034	775
IB_t	0	0	0	2809	775
IE_t	0	0	0	775	0
SB_t	0	561	2233	0	0
SE_t	561	2233	3286	0	0
Q_t				6095	

Objective value (total cost) = $22,846.49

MSM solution for scenario 1

t	1	2	3	4	5
d_t	561	1672	1053	2034	775
IB_t	0	0	1053	0	775
IE_t	0	0	0	0	0
SB_t	0	561	0	0	0
SE_t	561	2233	0	2034	0
Q_t			3286		2809

Objective value (total cost) = $23,287.73

GA solution for scenario 1

t	1	2	3	4	5
d_t	561	1672	1053	2034	775
IB_t	0	0	0	2809	775
IE_t	0	0	0	775	0
SB_t	0	561	2233	0	0
SE_t	561	2233	3286	0	0
Q_t				6095	

Objective value (total cost) = $22,846.49

MIP solution for scenario 2

t	1	2	3	4	5
d_t	367	2074	2041	628	962
IB_t	0	0	3631	1590	962
IE_t	0	0	1590	962	0
SB_t	0	367	0	0	0
SE_t	367	2441	0	0	0
Q_t			6072		

Objective value (total cost) = $22,661.21

MSM solution for scenario 2

t	1	2	3	4	5
d_t	367	2074	2041	628	962
IB_t	0	2074	0	1590	962
IE_t	0	0	0	962	0
SB_t	0	0	0	0	0
SE_t	367	0	2041	0	0
Q_t		2441		3631	

Objective value (total cost) = $23,119.10

GA solution for scenario 2

t	1	2	3	4	5
d_t	367	2074	2041	628	962
IB_t	0	0	3631	1590	962
IE_t	0	0	1590	962	0
SB_t	0	367	0	0	0
SE_t	367	2441	0	0	0
Q_t			6072		

Objective value (total cost) = $22,661.21

MIP solution for scenario 3

t	1	2	3	4	5
d_t	1524	1516	1695	631	1194
IB_t	0	0	3520	1825	1194
IE_t	0	0	1825	1194	0
SB_t	0	1524	0	0	0
SE_t	1524	3040	0	0	0
Q_t			6560		

Objective value (total cost) = $24,638.37

MSM solution for scenario 3

t	1	2	3	4	5
d_t	1524	1516	1695	631	1194
IB_t	0	1516	0	1825	1194
IE_t	0	0	0	1194	0
SB_t	0	0	0	0	0
SE_t	1524	0	1695	0	0
Q_t		3040		3520	

Objective value (total cost) = $24,911.44

GA solution for scenario 3

t	1	2	3	4	5
d_t	1524	1516	1695	631	1194
IB_t	0	0	3520	1825	1194
IE_t	0	0	1825	1194	0
SB_t	0	1524	0	0	0
SE_t	1524	3040	0	0	0
Q_t			6560		

Objective value (total cost) = $24,638.37

MIP solution for scenario 4

t	1	2	3	4	5
d_t	1131	832	769	682	1494
IB_t	0	0	2945	2176	1494
IE_t	0	0	2176	1494	0
SB_t	0	1131	0	0	0
SE_t	1131	1963	0	0	0
Q_t			4908		

Objective value (total cost) = $18,654.10

MSM solution for scenario 4

t	1	2	3	4	5
d_t	1131	832	769	682	1494
IB_t	0	1601	769	0	1494
IE_t	0	769	0	0	0
SB_t	0	0	0	0	0
SE_t	1131	0	0	682	0
Q_t		2732			2176

Objective value (total cost) = $18,964.76

GA solution for scenario 4

t	1	2	3	4	5
d_t	1131	832	769	682	1494
IB_t	0	0	0	2176	1494
IE_t	0	0	0	1494	0
SB_t	0	1131	1963	0	0
SE_t	1131	1963	2732	0	0
Q_t				4908	

Objective value (total cost) = $18,654.14

MIP solution for scenario 5

t	1	2	3	4	5
d_t	1281	789	1189	599	2016
IB_t	0	0	0	2615	2016
IE_t	0	0	0	2016	0
SB_t	0	1281	2070	0	0
SE_t	1281	2070	3259	0	0
Q_t				5874	

Objective value (total cost) = $22,243.01

MSM solution for scenario 5

t	1	2	3	4	5
d_t	1281	789	1189	599	2016
IB_t	0	1978	1189	0	2016
IE_t	0	1189	0	0	0
SB_t	0	0	0	0	0
SE_t	1281	0	0	599	0
Q_t		3259			2615

Objective value (total cost) = $22,530.68

GA solution for scenario 5

t	1	2	3	4	5
d_t	1281	789	1189	599	2016
IB_t	0	0	0	2615	2016
IE_t	0	0	0	2016	0
SB_t	0	1281	2070	0	0
SE_t	1281	2070	3259	0	0
Q_t				5874	

Objective value (total cost) = $22,243.01

MIP solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10
d_t	934	348	934	367	1886	537	566	725	1504	567
IB_t	0	0	0	2234	1867	0	0	0	2071	567
IE_t	0	0	0	1867	0	0	0	0	567	0
SB_t	0	934	1282	0	0	19	556	1122	0	0
SE_t	934	1282	2216	0	19	556	1122	1847	0	0
Q_t				4450					3918	

Objective value (total cost) = $31,681.18

MSM solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10
d_t	934	348	934	367	1886	537	566	725	1504	567
IB_t	0	1282	934	0	0	537	0	0	2071	567
IE_t	0	934	0	0	0	0	0	0	567	0
SB_t	0	0	0	0	367	0	0	566	0	0
SE_t	934	0	0	367	2253	0	566	1291	0	0
Q_t		2216				2790			3362	

Objective value (total cost) = $32,249.98

GA solution for scenario 6

t	1	2	3	4	5	6	7	8	9	10
d_t	934	348	934	367	1886	537	566	725	1504	567
IB_t	0	0	0	2253	1886	0	0	0	2071	567
IE_t	0	0	0	1886	0	0	0	0	567	0
SB_t	0	934	1282	0	0	0	537	1103	0	0
SE_t	934	1282	2216	0	0	537	1103	1828	0	0
Q_t				4469					3899	

Objective value (total cost) = $31,841.32

MIP solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10
d_t	1312	1071	1672	546	1233	1911	1418	1788	1254	1174
IB_t	0	0	2217	545	0	3316	1405	0	2428	1174
IE_t	0	0	545	0	0	1405	0	0	1174	0
SB_t	0	1312	0	0	1	0	0	13	0	0
SE_t	1312	2383	0	1	1234	0	13	1801	0	0
Q_t			4600			4550			4229	

Objective value (total cost) = $49,930.16

MSM solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10
d_t	1312	1071	1672	546	1233	1911	1418	1788	1254	1174
IB_t	0	0	1672	0	0	1911	0	0	1254	1174
IE_t	0	0	0	0	0	0	0	0	0	0
SB_t	0	1312	0	0	546	0	0	1418	0	0
SE_t	1312	2383	0	546	1779	0	1418	3206	0	0
Q_t			4055			3690			4460	1174

Objective value (total cost) = $50,416.45

GA solution for scenario 7

t	1	2	3	4	5	6	7	8	9	10
d_t	1312	1071	1672	546	1233	1911	1418	1788	1254	1174
IB_t	0	0	3451	1779	1233	0	0	4216	2428	1174
IE_t	0	0	1779	1233	0	0	0	2428	1174	0
SB_t	0	1312	0	0	0	0	1911	0	0	0
SE_t	1312	2383	0	0	0	1911	3329	0	0	0
Q_t			5834					7545		

Objective value (total cost) = $50,293.17

MIP solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10
d_t	1443	880	1026	1772	2026	1510	475	1102	1163	1934
IB_t	0	1907	1027	1	3554	1528	18	0	0	1934
IE_t	0	1027	1	0	1528	18	0	0	0	0
SB_t	0	0	0	0	0	0	0	457	1559	0
SE_t	1443	0	0	1771	0	0	457	1559	2722	0
Q_t		3350			5325					4656

Objective value (total cost) = $50,042.39

MSM solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10
d_t	1443	880	1026	1772	2026	1510	475	1102	1163	1934
IB_t	0	1906	1026	0	3536	1510	0	0	1163	1934
IE_t	0	1026	0	0	1510	0	0	0	0	0
SB_t	0	0	0	0	0	0	0	475	0	0
SE_t	1443	0	0	1772	0	0	475	1577	0	0
Q_t		3349			5308				2740	1934

Objective value (total cost) = \$50,623.14

GA solution for scenario 8

t	1	2	3	4	5	6	7	8	9	10
d_t	1443	880	1026	1772	2026	1510	475	1102	1163	1934
IB_t	0	1906	1026	0	3536	1510	0	0	0	1934
IE_t	0	1026	0	0	1510	0	0	0	0	0
SB_t	0	0	0	0	0	0	0	475	1577	0
SE_t	1443	0	0	1772	0	0	475	1577	2740	0
Q_t		3349			5308					4674

Objective value (total cost) = \$50,042.53

MIP solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10
d_t	1365	1922	1155	401	1610	1223	1466	401	481	1062
IB_t	0	0	3163	2008	1607	0	0	1944	1543	1062
IE_t	0	0	2008	1607	0	0	0	1543	1062	0
SB_t	0	1365	0	0	0	3	1226	0	0	0
SE_t	1365	3287	0	0	3	1226	2692	0	0	0
Q_t			6450					4636		

Objective value (total cost) = \$41,834.75

MSM solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10
d_t	1365	1922	1155	401	1610	1223	1466	401	481	1062
IB_t	0	3077	1155	0	0	1223	0	1944	1543	1062
IE_t	0	1155	0	0	0	0	0	1543	1062	0
SB_t	0	0	0	0	401	0	0	0	0	0
SE_t	1365	0	0	401	2011	0	1466	0	0	0
Q_t		4442				3234		3410		

Objective value (total cost) = \$41,968.46

GA solution for scenario 9

t	1	2	3	4	5	6	7	8	9	10
d_t	1365	1922	1155	401	1610	1223	1466	401	481	1062
IB_t	0	0	3166	2011	1610	0	0	1944	1543	1062
IE_t	0	0	2011	1610	0	0	0	1543	1062	0
SB_t	0	1365	0	0	0	0	1223	0	0	0
SE_t	1365	3287	0	0	0	1223	2689	0	0	0
Q_t			6453					4633		

Objective value (total cost) = $41,855.67

MIP solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10
d_t	1652	2071	2073	532	1660	1270	2070	1447	361	1920
IB_t	0	0	2077	4	0	3337	2067	0	0	1920
IE_t	0	0	4	0	0	2067	0	0	0	0
SB_t	0	1652	0	0	528	0	0	3	1450	0
SE_t	1652	3723	0	528	2188	0	3	1450	1811	0
Q_t			5800			5525				3731

Objective value (total cost) = $56,354.21

MSM solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10
d_t	1652	2071	2073	532	1660	1270	2070	1447	361	1920
IB_t	0	2071	0	2192	1660	0	2070	0	0	1920
IE_t	0	0	0	1660	0	0	0	0	0	0
SB_t	0	0	0	0	0	0	0	0	1447	0
SE_t	1652	0	2073	0	0	1270	0	1447	1808	0
Q_t		3723		4265			3340			3728

Objective value (total cost) = $56,731.89

GA solution for scenario 10

t	1	2	3	4	5	6	7	8	9	10
d_t	1652	2071	2073	532	1660	1270	2070	1447	361	1920
IB_t	0	0	2073	0	0	3340	2070	0	0	1920
IE_t	0	0	0	0	0	2070	0	0	0	0
SB_t	0	1652	0	0	532	0	0	0	1447	0
SE_t	1652	3723	0	532	2192	0	0	1447	1808	0
Q_t			5796			5532				3728

Objective value (total cost) = $56,374.84

Vitae

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3. Incubation Program EI KFUPM